# Working paper

# Hedging effectiveness of precious metals across frequencies: Evidence from Wavelet Based Dynamic Conditional Correlation Analysis

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**Abstract**

This study examines the dynamic relationship between precious metals and stock markets of major developed (G7) and emerging (BRICS) nations. We use a hybrid wavelet-based Dynamic Conditional Correlation (DCC) approach, which allows us to investigate the dynamic relationship between precious metals and stock markets in time and frequency domain. Our results suggest that DCC between the return series of precious metals and stock market varies with timescales in terms of dynamicity, persistence and strength of relationship. G7 and BRICS nations exhibit different dynamicity with precious metals over the study period (2000 to 2017). The dynamics between precious metals and G7 stock markets show similar patterns, which depicts a clustering behaviour, however, same is not true for BRICS nations. In contrast of existing literature, this study found that silver offers better hedging capability than other precious metals for both short and long-run. To construct a two-asset optimal portfolio of precious metal and stock index, palladium emerges as the most favourable option for both short and long run.

**Keywords:** Stock markets; Precious metals; G7, BRICS; DCC-GARCH; Wavelet

## 1. Introduction

The relationship between stock markets and precious metals has been of keen interest among researchers and practitioners alike. The primary interest of previous studies has been to explain investor's tendency to invest in other asset classes to diversify the risks associated with stock markets. Moreover, the economic turmoil and crisis of last decades have prompted investors to diversify the portfolio with alternative investment instruments like precious metals (Baur & McDermott, 2010; Bredin, Conlon, & Potì, 2015; Creti, Joëts, & Mignon, 2013; El Hedi Arouri, Lahiani, & Nguyen, 2015; Hillier, Draper, & Faff, 2006; Mensi, Hammoudeh, Reboredo, & Nguyen, 2014).

The interactions between different trading classes are considered to be an important factor in the resultant market dynamics (Gençay, Gradojevic, Selçuk, & Whitcher, 2010). The dynamics between stock markets and precious metals could impact the investor's investment behaviour, however, this behaviour seems to work differently in developed and developing markets (Baur & McDermott, 2010). Besides, investment horizon (long-run and short-run) also reported to play an important role in portfolio diversification strategies (Marshall, 1994). Furthermore, as argued by Baruník, Kočenda, & Vácha, (2016) such heterogeneity in market behaviour as well as interactions among different asset classes could result in dynamic relationship that may go unnoticed. Therefore, the objective of this study is to investigate the dynamic relationship between international stock markets (G7 and BRICS) with precious metals (gold, silver, platinum and palladium). More specifically, this paper investigates the dynamic relationship between stock markets and precious metals over different investment horizons.

The heterogeneity of market participants and investment choices emphasise the importance of analysing the time dependent co-movements. Moreover, The information transmission and spillover effect between different markets may also vary over different time horizons (Mensi, Hammoudeh, Shahzad, & Shahbaz, 2017). Active participants like financial institutions exploit the short-term movements in the markets, while passive investors like individual investors, insurance companies and non-financial firms' targets long-term performances of the investments. The preference of investment can thus be attributed to risk type of investors. Gold has emerged as a hedging instrument against abrupt movements in stock markets. Usage of gold as a hedging instrument has led to an increased interest in other precious metals, which serves a dual role of investment assets as well as industrial commodities (Sari, Hammoudeh, & Soytas, 2010).

Understanding the dynamic relationship between precious metals and stock markets is an important aspect for portfolio design and hedging strategies. However, these interactions have been examined to a limited extent. For example, Baur & Lucey (2010) analyse the time varying relationship between stock returns, bond returns and gold returns from 1995 to 2005 and reported that gold acts as a safe haven for stock markets of Germany, United Kingdom and United states. Authors also suggest that the safe haven property of gold is usually short lived and is predominant during economic shocks. Jain & Biswal (2016) in their study analyse the dynamic relationship between crude oil prices, gold prices, exchange rate and stock market for 2006 to 2015 in the case of India. Authors report the existence of dynamic relationship between gold prices and stock index. Interestingly, they argue that the fall in gold prices result in a fall in the stock market index in India.

Financialization of commodity markets has directed the interests of investors towards precious metals as an alternative investment instrument. As a result, it calls for further research on the co-movements between precious metals and stock markets. Moreover, analysing the dynamic co-movements of assets for different investment horizons could provide additional information and help market participants to predict price changes and policy makers to take financial stability measures (Baruník et al., 2016). To get the additional insights regarding the co-movements of precious metal prices and stock market indices, this study adopts the combination of two techniques first suggested by Lehkonen & Heimonen (2014). The wavelet-based dynamic conditional correlation analysis depicts the precious metals and stock markets interferences at different time horizons. The strength of the wavelet analysis to decipher hidden information has led to the increased usage of wavelet analysis to get deeper understanding of the underlying phenomenon in time-frequency domain. For instance, Baruník et al., (2016); Mensi, Hkiri, Al-Yahyaee, & Kang, (2018); Reboredo & Rivera-Castro, (2013); Vacha & Barunik, (2012); Das, Kannadhasan, Al-Yahyaee & Yoon (2018); Das, Bhowmik & Jana (2018) and Das & Kumar (2018) have employed wavelet analysis under different theoretical setups.

This study employs the multiresolution wavelet analysis to disentangle the precious metals and stock market dynamics at different time-horizons. Then, the results of wavelet analysis are used as an input in DCC-GARCH (dynamic conditional correlation- generalized autoregressive conditional heteroskedasticity) process. Due to its capability to capture the dynamic correlation, DCC has been used by number of authors in the past, for example, (Basher & Sadorsky, 2016; Creti, Ftiti, & Guesmi, 2014; Jones & Olson, 2013; Kim, Jung, & Qin, 2016; Klein, 2017). The combination of the above-mentioned approaches thus provides an opportunity for an in-depth examination of the dynamic correlation among the selected series. Moreover, the combination of these two techniques enable us to make inferences about the group of developed and emerging markets behaviour with respect to precious metals. Additionally, these techniques also allow to uncover the appropriate time-horizons to exploit the portfolio diversification benefits.

To the best of our knowledge, present study is the first to analyse the heterogeneity in correlations at different frequencies of precious metals and stock markets and therefore brings new insights to the existing body of literature. Our contribution to the literature is three-fold. First, this paper investigates the short-run and long-run dynamics between precious metals and stock markets. Due to the highly volatile properties of stock market and precious metal prices, any investigation is not complete without scrutinizing the time varying feature of these markets. It may happen that precious metals and stock markets do not exhibit any relationship in a short-run however, may show a strong relationship in a long-run. Segregating the data into various time scales (short run and long run) may also suggest the time periods to exploit the properties of financial assets. Second, this paper also investigates the appropriate hedging strategies. While literature suggests that gold provides a good hedge against adverse movements in stock exchanges however, is it true for other precious metals? Does investment in stock market can be used as a hedge against adverse movements in precious metals coupled with different time horizons? Third, this paper also considers the two-asset optimal portfolio in the case of stock market and precious metals over different time horizons.

Timescale analysis reveals some interesting dynamics between precious metals and stock markets. First, the dynamic correlation varies with timescales both in terms of dynamicity and strength of relationship. Second, developed and emerging markets exhibit different dynamic patterns over the study period. Both at returns level and higher timescales, developed countries follow up very closely to each other, which justify their clustering, but it is not true for BRICS nations. It suggests diversification benefits in the case of emerging nations over the long-time horizon. Third, on an average our results suggest that precious metals provide economical hedging against long positions in stock markets and not vice versa. Among precious metals, silver offers better hedging capability than other precious metals for both short and long-run. Fourth, in case of emerging markets portfolio of stock and precious metal should consist of higher levels of precious metals. However, to construct an optimal portfolio of stock markets and precious metals, palladium emerges as the most favourable option for both short and long run.

Rest of the paper is organized as follows. Section 2 describes the methodology employed for the study. Section 3 explains the data whereas section 4 provides empirical results. The hedging strategies are discussed in section 5. The section 6 discusses the portfolio weights. Section 7 provides the results of robustness tests. Finally, section 8 concludes.

## 2. Estimation Methodology

This section explains the methodology employed for studying the dynamic relationship between stock markets and precious metals.

### *2.1. Wavelet Multiresolution Analysis*

The wavelet-based technique is endowed with a superior ability over existing econometric methods to analyse time-series data in the time-frequency domain (Bouri, Gupta, Tiwari, & Roubaud, 2017; Lehkonen & Heimonen, 2014; Reboredo & Rivera-Castro, 2013). The wavelet transformation of a time-series into signals allows for localization of data in time and frequency. Wavelets are also capable of separating a signal into multi-horizon (multiresolution) components. Splitting up a signal by wavelet transformation technique can capture the finer details of a signal at smaller time-scales (Lehkonen & Heimonen, 2014). Moreover, wavelet-based technique is also capable to analyse the non-stationary signals.

According to Ramsey (2002) any function of time $f(t) \in L^2(R)$ can be represented as a sequence of projections by father $(\phi)$ and mother $(\psi)$ wavelets. The long-scale smooth components are represented by the father wavelets that integrate to one. Deviations from the smooth components are represented by mother wavelets, which integrate to zero. The scaling coefficients are generated by father wavelets, whereas the differencing coefficients are generated by mother wavelets.

The father wavelet is mathematically expressed as follows:

$$\phi_{j,k} = 2^{-j/2}\phi\left(\frac{t-2^j k}{2^j}\right) \; with \int \phi\,(t)dt = 1. \qquad \textbf{(1)}$$

whereas, the mother wavelet is defined as below:

$$\psi_{j,k} = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right) \; with \int \psi\,(t)dt = 0. \qquad \textbf{(2)}$$

where, $j$ and $k$ represent respective scale and translation parameters. The scale or the width of the functions $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ is measured by $2^j$.

The smooth coefficients obtained from the father wavelet are as follows:

$$s_{j,k} = \int f(t)\phi_{j,k}. \tag{3}$$

The detail coefficients of the mother wavelets are expressed as:

$$d_{j,k} = \int f(t)\psi_{j,k}. \qquad j = 1, \dots J \tag{4}$$

Thus, the wavelet representation of a function $f(t) \in L^2(R)$ is defined as the linear combination of wavelet function as below:

$$f(t) = \sum_k s_{j,k}\phi_{j,k}(t) + \sum_k d_{j,k}\psi_{j,k}(t) + \sum_k d_{j-1,k}\psi_{j-1,k}(t) + \dots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{5}$$

The eq. (5) may be simplified as:

$$f(t) = S_J + D_J + D_{J-1} + \dots D_j + \dots D_1 \tag{6}$$

The following projections are estimated by wavelet coefficient:

$$s_{j,k} \approx \int f(t)\phi_{j,k}(t)dt. \tag{7}$$

$$d_{j,k} \approx \int f(t)\psi_{j,k}(t)dt. \qquad j = 1, \dots J \tag{8}$$

where the basis function $\phi_{j,k}(t)$ and $\psi_{j,k}(t)$ are assumed to be orthogonal.

From eq. (6), $\{S_J, D_{J-1}, \dots D_j, \dots D_1\}$ is the resulting multiscale decomposition of $f(t)$. $D_j$ is the $j^{th}$ level wavelet detail corresponds to the changes in the series $\gamma_j$. The aggregated sum of variations at each detail scale is represented by $S_J$, which becomes smoother with higher levels of $j$ (Gencay, Selcuk, & Whitcher, 2002).

The maximum overlap discrete wavelet transform (MODWT) is applied to calculate the scale and wavelet coefficients. The decomposition of the time-series data under consideration was done using Daubechies (a family of orthogonal wavelets) least asymmetric filter of length eight [LA(8), hereafter]. LA(8) filters have at least couple of advantages over Haar wavelet filters. First, LA(8) filters are smoother (Gencay et al., 2002) and second, LA(8) filters generate uncorrelated coefficients across scales (Cornish, Bretherton, & Percival, 2006).

This study follows (Bouri et al., 2017) to decompose the stock market and precious metal series into $d_1$ to $d_3$. The resolutions of the data under investigations are provided for scales $2^j$ to $2^{j+1}$. The oscillation periods of 2-4, 4-8, 8-16, 16-32, 32-64 and 64-128 weeks (refer table 2) corresponds to wavelet scales $d_1$, $d_2$, $d_3$, $d_4$, $d_5$ and $d_6$ respectively. The smooth component $S_6$ represent movements in long-term.

*2.2. Dynamic Conditional Correlation*

DCC-GARCH came into existence with the works of Engle & Sheppard (2001) and Engle (2002). This method is suitable for analysing the contemporary conditional correlation between different return series, as it is the function of past correlations and the historical conditional volatilities. Further, the flexibility of DCC estimators is comparable to univariate GARCH models and also have a lower level of complexity in comparison to conventional GARCH models. DCC-GARCH model is based on the decomposition of the conditional covariance matrix into two-time varying parts: first, a conditional standard deviations matrix and second, a correlations matrix. The basic essence of the decomposition is to achieve an ease in estimation process by separating the univariate and multivariate dynamics.

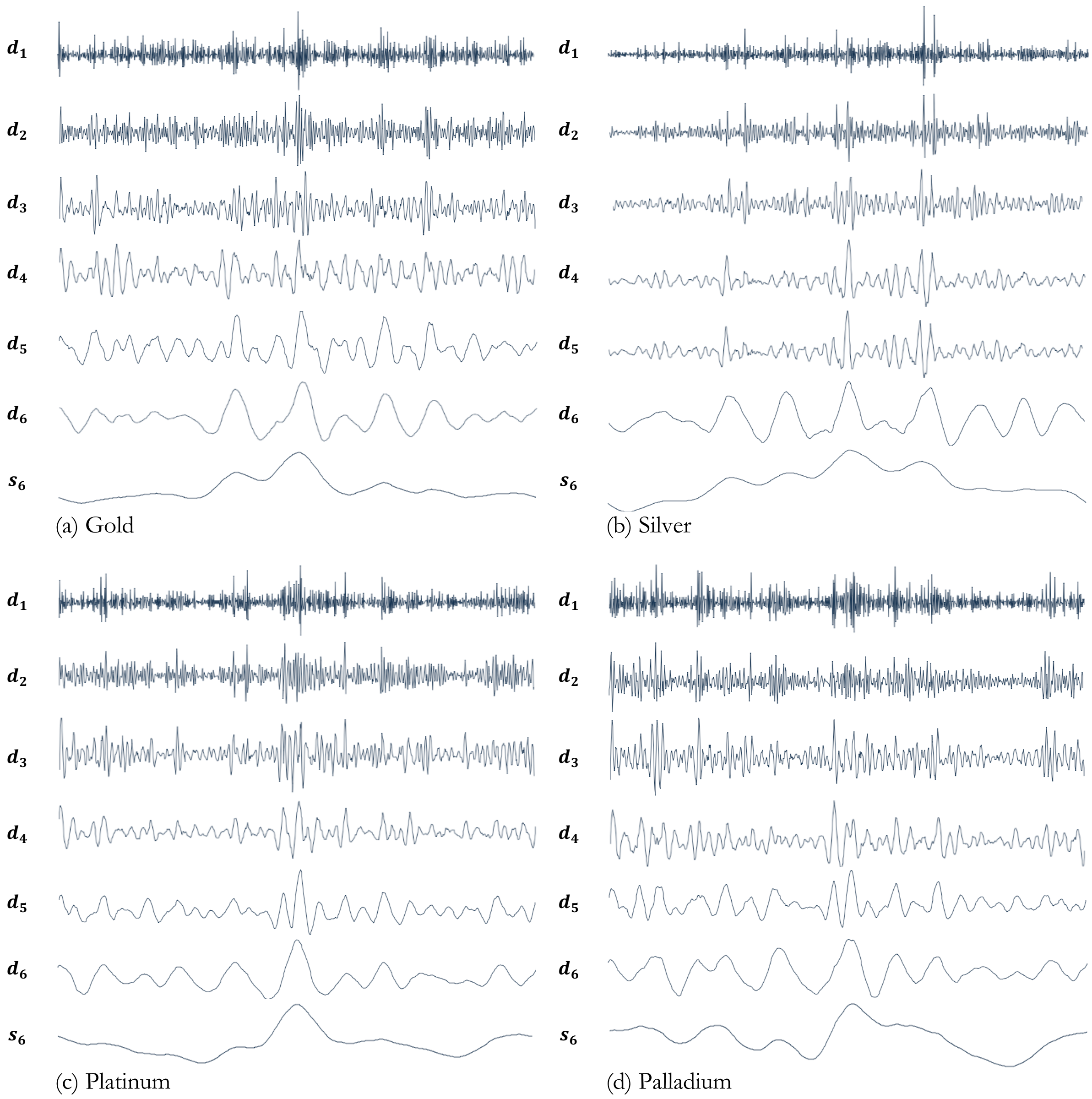


**Figure 1.** Multi-resolution analysis of precious metals group

**Table 1.** Time interpretation of different frequencies

| | | |
|---|---|---|
| $d_1$ | 2~4 weeks | Up to 1 Month |
| $d_2$ | 4~8 weeks | 1-2 Months |
| $d_3$ | 8~16 weeks | 2-4 Months |
| $d_4$ | 16~32 weeks | 4-8 Months |
| $d_5$ | 32~64 weeks | 8-16 Months |
| $d_6$ | 64~128 weeks | 16-32 Months |

The details of the DCC GARCH process is as follows:

$$H_t = D_t R_t D_t \qquad (9)$$

$H_t$ is a n × n matrix of conditional variances of mean-corrected n-return series $[r_t]$ at time $t$, where, $E[r_t] = 0$ and $Cov\ [r_t] = H_t$. $D_t$ is a n × n diagonal matrix of time varying standard deviations from n univariate GARCH models at time $t$.

$$D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & \cdots & 0 \\ 0 & \sqrt{h_{2t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{h_{nt}} \end{bmatrix} \qquad (10)$$

$\mathrm{R}_t$ is a time varying conditional correlation matrix of standardized disturbances $\varepsilon_{t,}$ where, $\varepsilon_t = D_t^{-1} r_t \sim N(0, R_t)$ and $\mathrm{R}_t$ is:

$$\mathrm{R}_t = \begin{bmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1n,t} \\ \rho_{12,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \rho_{n-1,n,t} \\ \rho_{1n,t} & \cdots & \rho_{n-1,n,t} & 1 \end{bmatrix} \qquad (11)$$

At least two criteria must satisfy in defining a DCC-GARCH process. First, $H_t$ is must be positive definite (as it is a covariance matrix). Since $D_t$ is a positive definite matrix because of its structure (positive diagonal elements), $\mathrm{R}_t$ has to a be positive definite matrix. Second, elements in $\mathrm{R}_t$ should be less than or equal to one. With these preconditions $\mathrm{R}_t$ can be decomposed into:

$$R_t = U_t^{*-1} U_t U_t^{*-1} \qquad (12)$$

$$U_t = (1 - \alpha - \beta)\overline{U} + \alpha \varepsilon_{t-1} \varepsilon_{t-1}^T + \beta U_{t-1} \qquad (13)$$

$U_t^*$ is a diagonal matrix with the elements consisting of square root of the diagonal elements of $U_t$.

$$U_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & \cdots & 0 \\ 0 & \sqrt{q_{22,t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{q_{nn,t}} \end{bmatrix} \qquad \textbf{(14)}$$

$U_t^*$ rescales the elements in $U_t$ such that $|\rho_{ij}| = \left|\frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}\right| \leq 1$. $U_t$ has to be positive definite so that $R_t$ is positive definite.

Where, $\overline{U} = Cov[\varepsilon_t \varepsilon_t^T] = E[\varepsilon_t \varepsilon_t^T]$ represents the unconditional covariance matrix of $\varepsilon_t$ (standardized errors) and $\overline{U}$ can be estimated as $\overline{U} = \frac{1}{T}\sum_{t=1}^{T} \varepsilon_t \varepsilon_t^T$.

Maximum likelihood estimation is used to estimate the model. The scalar parameters $\alpha$ and $\beta$ should meet certain conditions to ensure $H_t$ to be a positive definite matrix. First, $\alpha \geq 0, \beta \geq 0$ and second $\alpha + \beta < 1$. The DCC-GARCH results depend on these two parameters and can be stated as one of the strengths and weakness of the model. It is because of the requirement of two parameters estimation; the model is more likely to achieve an optimal solution.

The resultant correlation between two series depends on the standardized residuals. When the residuals move in the same direction, they push the correlation up and eventually with the passage of time coupled with complete information absorption, gradually fall back to the average level. On the other hand, when residuals move in opposite directions, it results in a low correlation. The time taken to move to different positions is governed by the parameters $\alpha$ and $\beta$. Specifically, $\alpha$ depicts the short-run persistence and $\beta$ indicates the long-run persistence.

**3. Data**

The study considers a period from January 01, 2000 to March 31, 2017 (899 observations), which is approximately symmetric to the global financial crisis episode of 2008. This study employed weekly data to avoid the problem of non-synchronous daily trading across the international stock and precious metals markets.[1] The national equity index[2] data for G7 and BRICS is used as stock data. As a proxy for precious-metals, dollar values of four metals i.e. gold, silver, platinum and palladium are considered. To overcome the issue of unit root, log returns series are used for the rest of the analysis.

$$X(t) = \log\left(\frac{S(t)}{S(t-1)}\right) \qquad (\mathbf{15})$$

where, $X(t)$ is the returns series and $S(t)$ denotes the selected time-series of precious metals or stock indices. The descriptive statistics of the returns series are presented in Table 2.

Table 2 shows that in the developed markets (G7), Germany yields the highest mean returns followed by US and Canada. On the other hand, the emerging markets (BRICS) perform relatively better than the developed counterparts, with Russia yielding the highest returns. In the case of precious metals, gold exhibits the highest returns followed by silver. The volatility in terms of standard deviation of returns, appears the highest for Germany and Italy in developed markets. In BRICS markets, Russia exhibits the maximum volatility followed by Brazil. In precious metals market, palladium shows the highest volatile behaviour followed by silver (refer Figure 2 for diagrammatic representation of annualized risk-return characteristics of national stocks and precious metals).

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[1] The true correlations between markets may be underestimated if non-synchronous data is used (Martens & Poon, 2001).

[2] Japan- Nikkei 225, US- SPX500, Germany- DAX, UK- FTSE 100, Italy- FTSE MIB, France- CAC40, Canada- S&P/TSX, Brazil- IBOV, Russia- MICEX index, India- S&P BSE SENSEX China- Shanghai Composite Index and South Africa- FTSE Top 40.

**Table 2.** Descriptive Statistics

| | Mean | SD | Skewness | Kurtosis | Jarque-Bera | Ljung-Box (10) Q-Statistic | |
|---|---|---|---|---|---|---|---|
| **G7** | | | | | | | |
| Japan | 4.29E-05 | 0.031 | -1.082 | 7.791 | 2420.080 | 0.17 | (0.685) |
| US | 0.0006 | 0.025 | -0.850 | 7.228 | 2039.728 | 4.30 | (0.038) |
| Germany | 0.0007 | 0.033 | -0.655 | 5.081 | 1017.922 | 1.31 | (0.252) |
| UK | 0.0001 | 0.025 | -1.048 | 11.657 | 5191.982 | 4.82 | (0.028) |
| Italy | -0.0007 | 0.033 | -0.889 | 6.138 | 1510.975 | 0.17 | (0.683) |
| France | -8.71E-05 | 0.030 | -0.870 | 6.284 | 1573.060 | 5.45 | (0.196) |
| Canada | 0.0006 | 0.024 | -0.954 | 7.211 | 2058.718 | 11.17 | (0.000) |
| **BRICS** | | | | | | | |
| Brazil | 0.0015 | 0.039 | -0.429 | 3.363 | 444.886 | 6.94 | (0.008) |
| Russia | 0.0026 | 0.045 | -0.205 | 7.159 | 1901.229 | 0.92 | (0.338) |
| India | 0.0019 | 0.032 | -0.549 | 3.139 | 408.5753 | 1.22 | (0.269) |
| China | 0.0008 | 0.034 | -0.195 | 2.326 | 204.8963 | 5.27 | (0.022) |
| South Africa | 0.0019 | 0.028 | -0.076 | 3.453 | 2039.728 | 5.16 | (0.023) |
| **Precious Metals** | | | | | | | |
| Gold | 0.0017 | 0.024 | -0.218 | 1.795 | 125.5184 | 0.12 | (0.732) |
| Silver | 0.0014 | 0.042 | -1.137 | 6.026 | 1535.773 | 2.53 | (0.111) |
| Platinum | 0.0009 | 0.031 | -0.528 | 2.661 | 302.6857 | 5.76 | (0.016) |
| Palladium | 0.0006 | 0.047 | -0.260 | 1.816 | 131.3625 | 7.63 | (0.005) |

Note: The critical value of Jarque-Bera (JB) test at 5% level is 5.99. Ljung-Box test was performed taking lag of 10. The p-values are reported in parentheses.

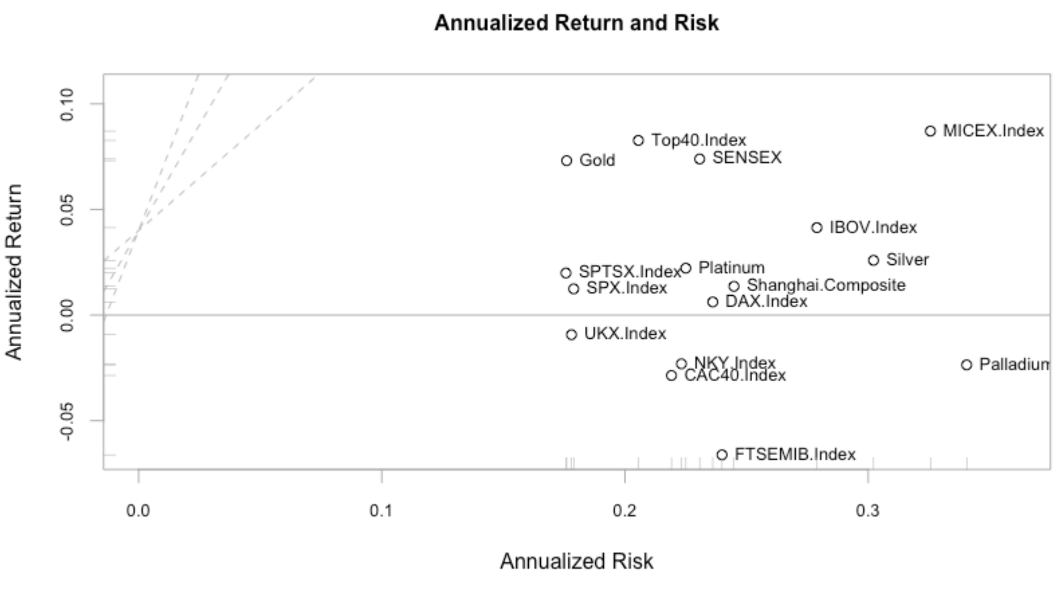


**Figure 2.** Annualized risk-return characteristics stocks and precious metals

The returns series for all G7, BRICS and precious metals index are negatively skewed and leptokurtic. The negative skewness coefficients suggest that negative returns are more frequent than large positive returns. In other word, all the markets are more susceptible to realize negative returns on any event of economic downturn (El Hedi Arouri, Jawadi, & Nguyen, 2010). The investor community often appreciates a significant and positive kurtosis coefficient. The underlying reason being a higher probability of positive returns. The returns series for all the assets clearly rejects the proposition of normality as reflected by the JB test results. The Ljung-Box Q-Statistic at order of 10 lags show serial dependence for Canada in developed markets and for Palladium in precious metals. Figure 3 shows significant cross-correlations between the markets, however gold shows relatively lower degrees of correlation with developed markets (Canada being the only exception). On the other hand, except for China, all other emerging markets exhibit higher degrees of correlations.

The non-normal distribution properties of the selected time-series therefore suggest the usage of wavelet and GARCH like models to analyse them. Furthermore, Andrieş, Ihnatov, & Tiwari (2016) argue that the linear correlation based techniques may not capture reliably the non-linear or frequency dependent linkages (if it exists) between two markets. Hence, a cautious interpretation of results is required to draw reasonable conclusions.

## 4. Empirical results

Empirical analysis was carried out in two steps. First, wavelet-based technique was employed to decompose the stock indices and precious metals returns series over different time horizons. In the second step, DCC-GARCH was used to investigate the dynamic correlation between decomposed series at respective frequencies and time horizons. For example, $d_1$ of gold was analysed with $d_1$ of Canadian stock index and $d_6$ of silver was considered with $d_6$ of Brazilian stock index. Similar pairing process was followed for other combinations of stock indices and precious metals. As a reference, this study also analyse the dynamic relationship between the level returns (R), which are non-decomposed series of stock indices and precious metals. Therefore, the final bivariate analysis included 288 pairs of stock index and precious metals in multiresolution framework and 48 pairs at R.

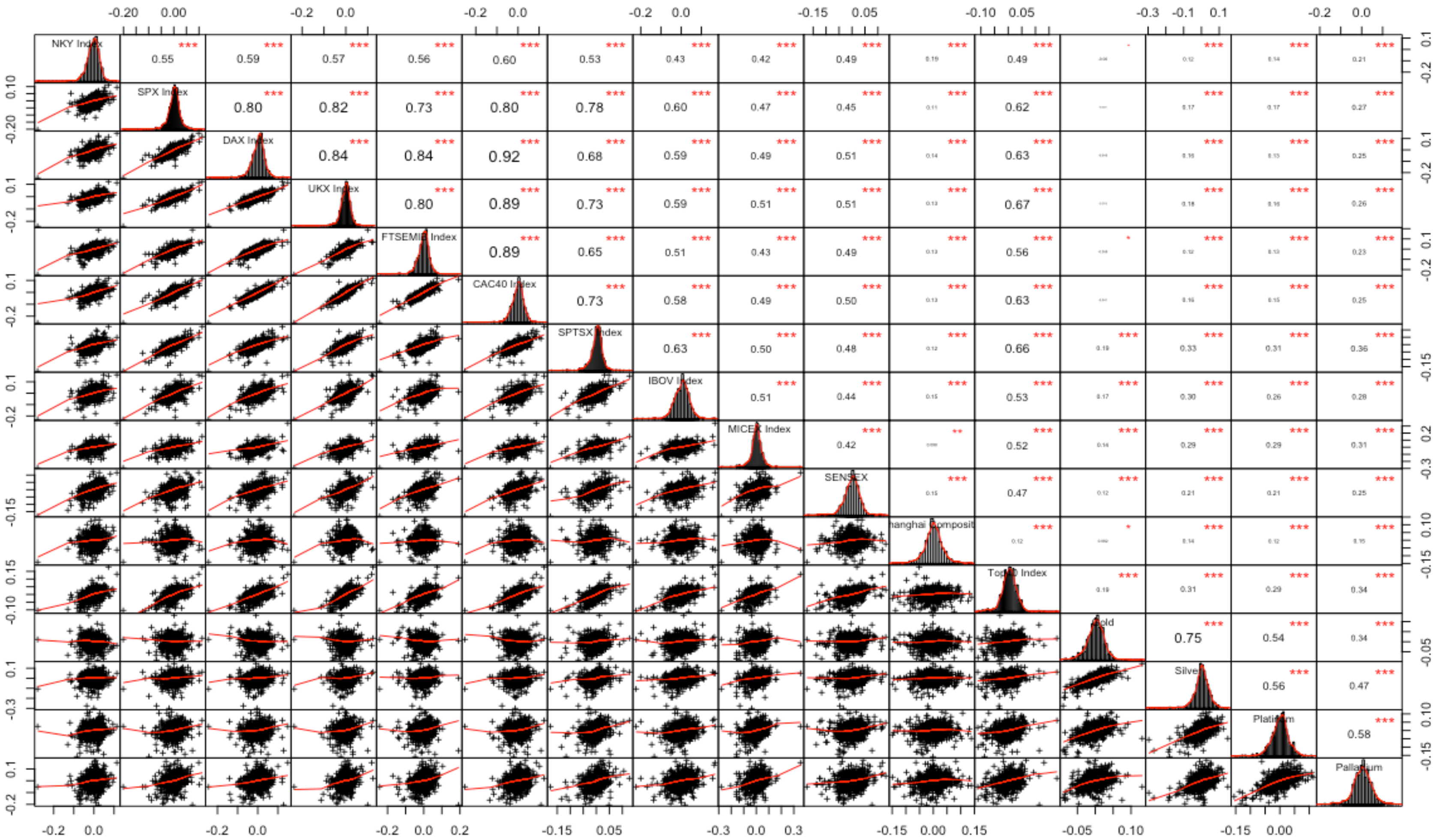

**Figure 3.** Scatter plots, histograms and Pearson correlation of index returns series

***Correlation is significant at the 0.01 level (2-tailed)

**Correlation is significant at the 0.05 level (2-tailed)

Table 3 reports the estimated parameters ($\alpha$ and $\beta$) of DCC-GARCH (1,1) bivariate framework for both level and decomposed analysis. The statistically significant parameters for almost all the series indicate a presence of dynamic correlation between precious metals and stock markets at R. All the estimated coefficients are non-negative and for all the coefficients $\alpha + \beta < 1$ indicating DCC movements around a constant level. It also indicates a mean reverting dynamic process. Moreover, high and significant $\beta$ values indicate long-run persistence. Interestingly, the estimated coefficients are not significant in the case of China for all precious metals, indicating some different dynamics working between precious metals and Chinese stock market in comparison to other nations. One of the possible reasons for the insignificant results could be because of precious metals are industrial commodities rather than investment instruments in case of China.

With regard to wavelet based DCC analysis ($d_1$ to $d_6$), parameter estimates for $d_1$ indicates that the statistical significance of the correlation breaks down in majority of markets in case of gold and silver and to some extent for platinum and palladium. It indicates precious metal returns and stock market returns are related to a very limited extent in short time horizons (2-4 weeks) and thus provide maximum diversification benefits. Our finding is strongly supported by earlier study of Baur and McDermott, (2010).

In case of $d_2$ and $d_3$ almost all the estimates are statistically significant with high $\beta$ values, which indicates high level of long-run persistence and a mean reverting process. The results are statistically significant even in the case of China, which indicates although dynamicity breaks in case of R but it exists for shorter time horizons ($d_2$ and $d_3$). For $d_2$, all the estimates are significant and for $d_3$, gold-US has zero and insignificant $\alpha$ parameter. Similarly, platinum-UK and palladium-China have zero $\beta$ parameters. Interestingly, the dynamicity breaks on several occasions from $d_4$ onwards as indicated by insignificant parameter estimates. Similarly, persistence of the relationship reduces drastically from $d_4$ onwards, where number of parameter estimates has small $\beta$ values.

Therefore, DCC results indicates that the relationship between precious metal returns and stock market returns persists during shorter time horizons and very little dynamicity and persistence exists during very short $d_1$ and long time periods ($d_4$, $d_5$, $d_6$).

Figure 4 to 6 shows the DCC between precious metals and stock indices for log R, and timescales $d_3$ and $d_6$ for G7 and BRICS nations from January 2000 to March 2017. To save space we have considered $d_3$ as a proxy for short-run and $d_6$ as a proxy for long-run[3]. Graphical representation of conditional correlation provides more details about the dynamic relationship between stock markets and precious metals. Figures show that dynamic correlations exist between precious metals and stock markets over the period of analysis. As reported in Table 3 and Figure 4 to 6 the dynamicity between precious metals and stock markets reduces considerably and correlation moves in clusters for higher timescales. On comparing the correlation between $d_3$ and $d_6$, it can be inferred that the diversification benefits tend to reduce with increase in timescale. All markets show similar DCC patterns in $d_6$.

*4.1 Developed vs. Emerging Markets*

Figure 4 to 6 suggest developed and emerging markets exhibit different co-movement patterns among the returns of precious metals and stocks markets. In case of developed markets DCC between precious metals and stock indices at R (Figure 4) move in a cluster except for Canada, which show higher correlation than other G7 countries. On the other hand, stock markets of Japan and Italy are least correlated with precious metals. DCC between gold and G7 stock indices show negative correlations for maximum duration, except for Canada. This shows an important input for portfolio diversification strategies. In case of BRICS nations, the co-movement is also similar but show more wide-ranging patterns. Brazil and South Africa show high DCC values while China show low correlation among emerging countries. DCC between precious metals and stock markets

[3] DCC graphs for d1, d2, d4 and d5 are not shown and are available upon request to authors.

show a sharp decline in correlation during recent financial crisis show the movement in opposite directions for commodity and equity markets. This supports the earlier findings that investors divert their funds to precious metal markets whenever economic shocks hit the equity market investors.

Figure 5 and 6 show the DCC between precious metals and stock markets at $d_3$ and $d_6$ timescales. Table 3 and Figure 4 indicate that $d_3$ exhibits a persistent and mean reverting process while $d_6$ shows low level of persistence. DCC values of $d_3$ (G7) show high variability and range in comparison to R. For example, high and low DCC values for R (G7) ranged from approximately 0.3 to -0.2 whereas $d_1$ values ranged from approximately 0.8 to -0.45. It indicates that for small time durations DCC among precious metals and stock markets exhibit high volatility and provide high return and high-risk scenarios but persist for small durations. On the other hand, the dynamic correlation between precious metals and stock returns increase for $d_6$ (G7) but the process loses the portfolio diversification advantages because of increase in homogeneity of member countries. In the case of BRICS nations, the portfolio diversification advantages exist during $d_3$ and continue for $d_6$ (figure 6). Therefore, BRICS nations provide better opportunities for portfolio diversification strategies.

Overall these results suggest that there is some degree of homogeneity between the G7 nations however BRICS nations fail to exhibit such attribute. We report Canada as an anomaly among G7 nations with respect to precious metals and stock market dynamics. One of the plausible reasons could be that Canadian stock index consists more commodity companies in comparison to other G7 nations. DCC without wavelet segregation, i.e. at R, failed to provide any statistically significant result in case of China. However, DCC results for China were found to be statistically significant at various time horizons. This result indicates the advantage of employing timescale analysis. Results also indicated that diversification opportunities reduce with increase in timescale. This could be because investors tend to exploit any arbitrage opportunity that exists in the market. Moreover, over the period of time (long run) there are no hedging opportunities available to the investors. Investors who fail to adjust their investment positions within short time (4-16 weeks) span may suffer losses during the long run as dynamic relationship between stock market and precious metals has lower persistent levels in the long run. Although, wavelet based DCC could reveal interesting results, however, DCC at R (without wavelet decomposition) could show that dynamic correlation between stock market and precious metals reduced significantly during the recent financial crisis (2007-2009). Though, this pattern is not very noticeable in short or long run. Nevertheless, this result indicates that a significant reduction in DCC between stock markets and precious metals could indicate an onset of economic downturn and investors should make adjustments to their portfolio accordingly.[4,5]

## 5. Hedging

Kroner & Sultan, 1993 argues that hedge ratios may be constructed by using the conditional volatility estimates. An asset $(x)$ on which a long-position is taken may be hedged with a second asset $(y)$ by taking a short position. Thus, the hedge ratio between the two assets may be represented as:

$$\beta_{xy,t} = \frac{h_{xy,t}}{h_{yy,t}} \qquad (16)$$

$\beta_{xy,t}$ is the hedge ratio between asset $x$ and $y$ at time $t$, $h_{xy,t}$ is the conditional covariance between asset $x$ and $y$ at time $t$, $h_{yy,t}$ is the conditional variance of asset $y$. Table 4 reports the average

[4] We also checked the robustness of our results by analyzing the level and wavelet decomposed series ($d_1$ to $d_6$) using the Generalized Orthogonal GARCH test (GO-GARCH) model proposed by Van Der Weide, 2002. Our findings were consistent with the change in methodology, which suggests the robustness of our findings. The results for log returns series and wavelet-decomposed values at different time-scales for GO-GARCH models are available upon request to authors.

[5] One of the anonymous reviewers suggested to investigate the asymmetric dynamic conditional correlation (ADCC) between stock market and precious metals, however we did not find asymmetric parameter to be significant in case of R and from $d_1$ to $d_6$. Therefore, ADCC results are not shown here, however are available upon request.

hedge ratios of US and Indian stock market indices with precious metals at R, $d_1$ and $d_6$.[6][7] For example the average value of hedge ratio between gold and US at R is 0.02, and for gold and India is 0.08. These results indicate that $1 long position in gold can be hedged for 2 cents with a short position in SPX500 (US). Similarly, a $1 long position in gold can be hedged with a short position for 8 cents in Sensex (India). From the table 4 it can be calculated the cheapest hedge at R in the case of US is a long position in SPX500 and short position in gold and for India is long position in gold and short position in Sensex. Notice that on an average hedge ratio decreases from R to $d_1$ and then again increases for $d_6$.

Figure 7 to 9 reports the hedging ratios between US and precious metals and also between India and precious metals for the sample period (January 2000 to March 2017)[8]. For log returns, the hedge ratio between US and precious metals show high variability after 2008, which usually peaked around 2011-2012. Gold and US hedge ratios show high variability in values even before 2008. Hedging ratios for gold and Indian stock market shows considerably different movements to US and gold hedging ratios, which seemed to move quite close over the sample period. Variability in hedging ratios in case of Gold and India also increased after 2008 but not as in the case of Gold and US. Notice that in the case of log returns hedging ratio of precious metals with US stock market index record values in excess of unity, except for gold and platinum and same is not true in the case of hedging ratios of gold and Indian stock market index. On the other hand, the hedging ratios vary considerably in $d_3$ and $d_6$. There is abnormal high level of hedging ratio at the beginning of sample period in the case of India and precious metals ($d_6$), which also falls abnormally. While US and precious metal ratio show similar variability in $d_3$ and $d_6$ it is not true in the case of India and precious metals. Although as shown in Table 4 on an average hedging

[6] Result for other R series and frequencies of nations and precious are not shown and are available upon request.
[7] Author is aware of the fact that there exists high level of heterogeneity among BRICS countries but to save space this study took US as a proxy for developed market and India as a proxy for developing market and continue with the selection for the next section.
[8] Figures for other log return series and frequencies of countries and precious metals are available upon request.

ratio decreased from R to $d_1$ but at the same time Figure 8 shows increase in variation or risk. Same pattern continues for US and precious metals at $d_6$, but due to very high hedging ratios pushing mean values on higher side. Interestingly, silver offers better hedging capability than other precious metals for both short and long-run. On an average our results suggest that precious metals provide better (less costly) hedge against long positions in stock markets rather other way around.

**6. Portfolio weights**

Kroner & Ng (1998) suggest that DCC GARCH (multiple GARCH model) volatility estimates can be used to construct optimal portfolio weights.

$$w_{xy,t} = \frac{h_{yy,t} - h_{xy,t}}{h_{xx,t} - 2h_{xy,t} + h_{yy,t}}$$

$$w_{xy,t} = \begin{cases} 0, if\ w_{xy,t} < 0 \\ w_{xy,t}, if 0 \leq w_{xy,t} \leq 1 \\ 1, if\ w_{xy,t} > 1 \end{cases}$$

$w_{xy,t}$ is the weight of asset $x$ in a one-dollar portfolio of $x$ and $y$, at time $t$. $h_{xy,t}$ is the conditional covariance between asset $x$ and $y$ at time $t$, $h_{yy,t}$ is the conditional variance of asset $y$. The weight of asset $y$ at time $t$ is $1 - w_{xy,t}$. Table 5 reports the summary statistics of portfolio weights computed from DCC model results for precious metals with US index and precious metals with Indian index[9]. The average weight for the Gold/US portfolio is 0.44, indicates that for a $1 portfolio, 44 cents should be invested in Gold and 56 cents should be invested in SPX500. Portfolio weights show different pattern for US and India. It can be noticed in Table 5 that portfolio weights for US increases from R to $d_3$ to $d_6$, indicates a requirement for higher investment in precious metals in long time horizons. On the other hand, in case of India investor should invest more in precious metals in short to medium term than in long time horizons. These results indicate that on an average portfolio of precious metals and stock index should include

[9] Result for other log return series and frequencies ($d_1$ to $d_6$) of countries and precious are not shown and are available upon request.

higher levels of precious metals in emerging than in advanced nations. However, among precious metals palladium emerges as the most favourable option for both short and long-run.

## 7. Conclusion

This study investigates the dynamic relationship between precious metals and major stock market indices (G7 and BRICS) over different time horizons from 2000 to 2017 to capture the potential effects of market type on the co-movement dynamics. This study also investigates the hedging characteristic of precious metals and stock markets. Further, optimal portfolios were also constructed to examine the ideal weights of precious metal and stock market index in case of two asset portfolio. Results suggest that the usage of time varying estimation to model the dynamics of precious metals and stock markets provide additional information regarding their behaviour in short and long-run.

Wavelet based DCC approach was employed to carry out the analysis. First, the decomposed return series of precious metals and stock market indices were obtained and then DCC-GARCH was used to analyse the dynamics between them. Wavelet analysis segregated the return series into different time horizons. Whereas, DCC captured the time-varying relationship between precious metals and stock markets returns. Therefore, combination of techniques enhanced the capability of DCC to analyse the time varying correlation by adding the timescale dynamics. Due to better capability of wavelet transformation to capture the finer details, this study is able to provide some interesting results regarding co-movements of precious metals and stock markets. For example, in case of China, return level (R) DCC failed to capture the dynamic relationship between precious metals and Chinese stock market. However, wavelet decomposition revealed a dynamic relationship between Chinese stock market and precious metals over different time horizons. Furthermore, the combined approaches allowed to capture the variations between and within developed and emerging markets in addition of capturing the difference between short-run and long-run time horizons.

Results suggest an opportunity for international portfolio diversification for both within and between G7 and BRICS nations. DCC between advanced nations (G7) and precious metals move in clusters except for Canada, while found higher variations in case of emerging nations (BRICS). In line with Lehkonen and Heimonen (Lehkonen & Heimonen, 2014), results also suggest that BRICS nations should not be clustered in one group. Results also suggest that G7 nations provide fewer opportunities for diversification within in comparison to BRICS nations. Moreover, it appears that investors divert their funds to precious metal markets whenever economic shocks hit the equity market investors. While analysing the multidimensional wavelet results using DCC, findings suggest that correlation structure varies significantly between different timescales and markets. For higher timescales, the variations in the co-movements of precious metals and stock markets reduce and co-movements tend to move together. This phenomenon is more pronounced in the case of advanced nations in comparison to emerging markets. Overall, in higher timescales, developed markets provide very low portfolio diversification opportunities.

Portfolio analysis provides additional details of the dynamics between stock markets and precious metals. Hedging ratios suggest on an average lower hedge ratio during short run and higher hedge ratios in the long run. The variation in hedging ratios also increase at higher timescales which suggest that investment risk increases over the longer time horizons. More interestingly, precious metals provide better and economical way of hedging against stock markets than other way around. In contrast of existing literature, this study found that silver offers better hedging capability than other precious metals for both short and long-run. Investigation of portfolio weights suggest that investor should include higher levels of precious metals in their stock market and precious metal portfolio in case of developed markets at higher timescales. On the other hand, investors should invest in higher level of precious metals in short and medium run than in long time horizons. On an average, in the case of emerging markets the level of investment in precious metals in a portfolio of stock market index and precious metals should be higher than that of advanced nations. To construct a two-asset optimal portfolio of precious metal and stock index, palladium emerges as the most favourable option for both short and long run.

These findings suggest that the usage of time and frequency analysis improve our understanding of short and long run market dynamics. The time scale analysis between precious metals and stock markets indicate that on an average advanced nation share similar dynamics in long-run. Portfolio analysis at different time scales suggests different investment strategies for type of nation and timescale. The dynamic causality between precious metals and stock markets at different timescales can be a possible future research avenue.

These findings have some significant economic implications. First, investors should not only focus on the returns but should also investigate the dynamics between asset classes over the different time horizons. Such timely adjustment in portfolio allocation can safeguard the interests of the investors. Second, China appears to be an anomaly in the group of stock markets investigated in this study. It could be because of tight regulations, however due to its significant role in world economy such behaviour could moderate or cease to exist in future. Third, in contrast to general perception, silver could play a major more role in risk reduction or portfolio diversification. However, it requires further investigation to acknowledge the hedging properties of silver in comparison to gold and investors will need a significant evidence to replace the gold as an ideal hedging asset. Fourth, with financialization of commodities, other commodities like palladium can provide an economical way to construct an optimal portfolio. Fifth, an extended period of higher dynamic correlation between stock markets and precious metals followed by a significant decline can signal economic downturn. Furthermore, economists, policy makers and investors should not overlook the time varying and contagion nature of the relationship between stock markets and precious metals while making investment or policy related decisions. Such considerations may help in mitigating the destabilizing impact of shocks to stock markets.

Nonetheless, the authors must acknowledge that this study suffers from certain limitations, which paves the way for future research. For instance, this study does not examine the hedging performance in extreme market conditions (i.e. safe-haven proposition) across different time-scales. Since, financial markets are dynamic in nature, some of the previous studies use daily frequency in their respective analysis (*see* Shik Lee, H. (2004); Madaleno, M., & Pinho, C. (2012); Aloui, C., & Hkiri, B. (2014); Marfatia, H. A. (2017) beside others). However, to address the issue of non-synchronous data, this study considers weekly returns and a future study with daily data might enrich our understanding on the relationship between stock markets and precious metals[10]. Moreover, the other contemporary investment instruments such as Cryptocurrencies may also be included in the asset basket.

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[10] The authors are thankful to the reviewer for highlighting this limitation of the study.

**Table 3.** Estimated pair-wise DCC parameters

| | Gold | | | | | Silver | | | | | Platinum | | | | | Palladium | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | α | Sig | B | Sig | α+β | A | Sig | β | Sig | α+β | α | Sig | β | Sig | α+β | α | Sig | β | Sig | α+β |
| **R** | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.0213 | 0.013# | 0.9731 | 0.000$ | 0.9943 | 0.0198 | 0.021# | 0.9709 | 0.000$ | 0.9907 | 0.0216 | 0.002$ | 0.9693 | 0.000$ | 0.9908 | 0.0090 | 0.086* | 0.9839 | 0.000$ | 0.9929 |
| US | 0.0349 | 0.005$ | 0.9331 | 0.000$ | 0.9680 | 0.0304 | 0.004$ | 0.9397 | 0.000$ | 0.9701 | 0.0250 | 0.000$ | 0.9603 | 0.000$ | 0.9852 | 0.0187 | 0.075* | 0.9761 | 0.000$ | 0.9947 |
| Germany | 0.0360 | 0.007$ | 0.9336 | 0.000$ | 0.9696 | 0.0242 | 0.010# | 0.9527 | 0.000$ | 0.9769 | 0.0184 | 0.007$ | 0.9695 | 0.000$ | 0.9879 | 0.0167 | 0.002$ | 0.9787 | 0.000$ | 0.9954 |
| UK | 0.0192 | 0.197 | 0.9500 | 0.000$ | 0.9692 | 0.0193 | 0.020# | 0.9607 | 0.000$ | 0.9800 | 0.0122 | 0.006$ | 0.9791 | 0.000$ | 0.9913 | 0.0146 | 0.000$ | 0.9831 | 0.000$ | 0.9977 |
| Italy | 0.0206 | 0.055* | 0.9545 | 0.000$ | 0.9751 | 0.0205 | 0.076* | 0.9507 | 0.000$ | 0.9712 | 0.0142 | 0.017# | 0.9753 | 0.000$ | 0.9895 | 0.0195 | 0.164 | 0.9712 | 0.000$ | 0.9907 |
| France | 0.0311 | 0.022# | 0.9385 | 0.000$ | 0.9696 | 0.0295 | 0.009$ | 0.9424 | 0.000$ | 0.9719 | 0.0167 | 0.010# | 0.9707 | 0.000$ | 0.9874 | 0.0211 | 0.093* | 0.9731 | 0.000$ | 0.9942 |
| Canada | 0.0323 | 0.009$ | 0.9458 | 0.000$ | 0.9782 | 0.0466 | 0.007$ | 0.9357 | 0.000$ | 0.9823 | 0.0207 | 0.003$ | 0.9748 | 0.000$ | 0.9955 | 0.0182 | 0.005$ | 0.9791 | 0.000$ | 0.9973 |
| Brazil | 0.0344 | 0.334 | 0.9245 | 0.000$ | 0.9589 | 0.0302 | 0.064* | 0.9460 | 0.000$ | 0.9762 | 0.0187 | 0.001$ | 0.9768 | 0.000$ | 0.9955 | 0.0207 | 0.030# | 0.9756 | 0.000$ | 0.9963 |
| Russia | 0.0301 | 0.291 | 0.7953 | 0.002$ | 0.8254 | 0.0128 | 0.055* | 0.9775 | 0.000$ | 0.9903 | 0.0177 | 0.003$ | 0.9755 | 0.000$ | 0.9932 | 0.0181 | 0.004$ | 0.9772 | 0.000$ | 0.9953 |
| India | 0.0526 | 0.034# | 0.8711 | 0.000$ | 0.9238 | 0.0255 | 0.071* | 0.9541 | 0.000$ | 0.9796 | 0.0180 | 0.009$ | 0.9747 | 0.000$ | 0.9927 | 0.0104 | 0.012# | 0.9874 | 0.000$ | 0.9977 |
| China | 0.0092 | 0.661 | 0.7605 | 0.000$ | 0.7697 | 0.0432 | 0.282 | 0.6053 | 0.000$ | 0.6485 | 0.0221 | 0.389 | 0.8514 | 0.000$ | 0.8736 | 0.0150 | 0.154 | 0.9535 | 0.000$ | 0.9684 |
| SA | 0.0158 | 0.097* | 0.9672 | 0.000$ | 0.9830 | 0.0353 | 0.035# | 0.9429 | 0.000$ | 0.9782 | 0.0192 | 0.000$ | 0.9773 | 0.000$ | 0.9965 | 0.0161 | 0.000$ | 0.9826 | 0.000$ | 0.9987 |
| $d_1$ | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.2058 | 0.000$ | 0.3552 | 0.000$ | 0.5610 | 0.0914 | 0.011# | 0.2784 | 0.100 | 0.3698 | 0.1167 | 0.009$ | 0.6788 | 0.000$ | 0.7955 | 0.0019 | 0.581 | 0.9929 | 0.000$ | 0.9948 |
| US | 0.0984 | 0.010# | 0.0000 | 0.999 | 0.0984 | 0.0772 | 0.037# | 0.4294 | 0.007$ | 0.5066 | 0.0139 | 0.422 | 0.9502 | 0.000$ | 0.9641 | 0.0300 | 0.023# | 0.9444 | 0.000$ | 0.9744 |
| Germany | 0.3320 | 0.000$ | 0.0000 | 0.999 | 0.3320 | 0.0000 | 0.999 | 0.9087 | 0.000$ | 0.9087 | 0.3685 | 0.000$ | 0.0000 | 0.999 | 0.3685 | 0.2159 | 0.000$ | 0.2227 | 0.262 | 0.4386 |
| UK | 0.0677 | 0.142 | 0.2628 | 0.219 | 0.3305 | 0.0897 | 0.044# | 0.4190 | 0.000$ | 0.5087 | 0.1371 | 0.690 | 0.0000 | 0.999 | 0.1371 | 0.2812 | 0.000$ | 0.1544 | 0.082* | 0.4357 |
| Italy | 0.0763 | 0.044# | 0.6732 | 0.000$ | 0.7495 | 0.3074 | 0.226 | 0.0000 | 0.999 | 0.3074 | 0.1212 | 0.030# | 0.8788 | 0.000$ | 1.0000 | 0.1008 | 0.006$ | 0.6166 | 0.000$ | 0.7174 |
| France | 0.1483 | 0.000$ | 0.0051 | 0.963 | 0.1534 | 0.1710 | 0.006$ | 0.0909 | 0.371 | 0.2619 | 0.0103 | 0.504 | 0.9362 | 0.000$ | 0.9466 | 0.0949 | 0.033# | 0.7226 | 0.000$ | 0.8174 |
| Canada | 0.1006 | 0.002$ | 0.7208 | 0.000$ | 0.8214 | 0.1790 | 0.820 | 0.0000 | 0.999 | 0.1790 | 0.2145 | 0.000$ | 0.4043 | 0.024# | 0.6188 | 0.1790 | 0.820 | 0.0000 | 0.999 | 0.1790 |
| Brazil | 0.0801 | 0.047# | 0.0000 | 0.999 | 0.0801 | 0.3046 | 0.000$ | 0.0747 | 0.595 | 0.3793 | 0.1692 | 0.000$ | 0.3462 | 0.004$ | 0.5154 | 0.2330 | 0.000$ | 0.3184 | 0.005$ | 0.5513 |
| Russia | 0.1600 | 0.000$ | 0.1334 | 0.105 | 0.2933 | 0.2450 | 0.000$ | 0.0852 | 0.250 | 0.3302 | 0.0737 | 0.037# | 0.5042 | 0.000$ | 0.5779 | 0.1157 | 0.002$ | 0.5969 | 0.000$ | 0.7126 |
| India | 0.2251 | 0.000$ | 0.1496 | 0.132 | 0.3747 | 0.0912 | 0.013# | 0.0001 | 0.999 | 0.0913 | 0.0299 | 0.293 | 0.7077 | 0.003$ | 0.7376 | 0.1931 | 0.000$ | 0.0000 | 0.999 | 0.1931 |
| China | 0.1778 | 0.085* | 0.0000 | 0.999 | 0.1778 | 0.2883 | 0.000$ | 0.2366 | 0.001$ | 0.5249 | 0.1693 | 0.000$ | 0.4420 | 0.000$ | 0.6113 | 0.0058 | 0.705 | 0.8795 | 0.000$ | 0.8853 |
| SA | 0.1777 | 0.012# | 0.2727 | 0.087* | 0.4504 | 0.0218 | 0.562 | 0.9571 | 0.000$ | 0.9790 | 0.4395 | 0.000$ | 0.0000 | 0.999 | 0.4395 | 0.3982 | 0.000$ | 0.0853 | 0.348 | 0.4835 |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{d_2}$ | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.1571 | 0.000$ | 0.6479 | 0.000$ | 0.8051 | 0.1313 | 0.000$ | 0.6416 | 0.000$ | 0.7729 | 0.1823 | 0.000$ | 0.5779 | 0.000$ | 0.7602 | 0.1470 | 0.000$ | 0.5942 | 0.000$ | 0.7412 |
| US | 0.2101 | 0.000$ | 0.5137 | 0.000$ | 0.7238 | 0.1929 | 0.000$ | 0.6123 | 0.000$ | 0.8052 | 0.1340 | 0.000$ | 0.7234 | 0.000$ | 0.8575 | 0.1140 | 0.000$ | 0.7242 | 0.000$ | 0.8383 |
| Germany | 0.1747 | 0.000$ | 0.6718 | 0.000$ | 0.8465 | 0.1905 | 0.000$ | 0.5433 | 0.000$ | 0.7337 | 0.1264 | 0.000$ | 0.6787 | 0.000$ | 0.8052 | 0.1666 | 0.000$ | 0.6584 | 0.000$ | 0.8250 |
| UK | 0.1290 | 0.000$ | 0.5545 | 0.000$ | 0.6835 | 0.1103 | 0.007$ | 0.5753 | 0.000$ | 0.6856 | 0.1025 | 0.002$ | 0.6991 | 0.000$ | 0.8016 | 0.2015 | 0.000$ | 0.5856 | 0.000$ | 0.7871 |
| Italy | 0.1202 | 0.004$ | 0.6852 | 0.000$ | 0.8053 | 0.2250 | 0.000$ | 0.5105 | 0.000$ | 0.7356 | 0.1902 | 0.000$ | 0.6168 | 0.000$ | 0.8071 | 0.1737 | 0.000$ | 0.5778 | 0.000$ | 0.7515 |
| France | 0.1808 | 0.000$ | 0.6310 | 0.000$ | 0.8118 | 0.1173 | 0.000$ | 0.6434 | 0.000$ | 0.7607 | 0.1634 | 0.000$ | 0.6863 | 0.000$ | 0.8498 | 0.1644 | 0.000$ | 0.6026 | 0.000$ | 0.7669 |
| Canada | 0.1269 | 0.000$ | 0.6677 | 0.000$ | 0.7946 | 0.2450 | 0.000 | 0.0852 | 0.250 | 0.3302 | 0.1739 | 0.000$ | 0.6701 | 0.000$ | 0.8439 | 0.1430 | 0.000$ | 0.7060 | 0.000$ | 0.8490 |
| Brazil | 0.1883 | 0.000$ | 0.5729 | 0.000$ | 0.7611 | 0.1442 | 0.000$ | 0.6181 | 0.000$ | 0.7624 | 0.1980 | 0.000$ | 0.5891 | 0.000$ | 0.7871 | 0.1656 | 0.000$ | 0.6980 | 0.000$ | 0.8636 |
| Russia | 0.1990 | 0.000$ | 0.6119 | 0.000$ | 0.8109 | 0.2050 | 0.000$ | 0.5454 | 0.000$ | 0.7504 | 0.2100 | 0.000$ | 0.5699 | 0.000$ | 0.7799 | 0.1930 | 0.000$ | 0.5785 | 0.000$ | 0.7714 |
| India | 0.2500 | 0.000$ | 0.5239 | 0.000$ | 0.7738 | 0.1629 | 0.000$ | 0.6017 | 0.000$ | 0.7646 | 0.1149 | 0.000$ | 0.6684 | 0.000$ | 0.7834 | 0.1821 | 0.000$ | 0.6313 | 0.000$ | 0.8134 |
| China | 0.1933 | 0.000$ | 0.5726 | 0.000$ | 0.7659 | 0.0766 | 0.019# | 0.6649 | 0.000$ | 0.7415 | 0.1585 | 0.000$ | 0.6463 | 0.000$ | 0.8048 | 0.1235 | 0.007$ | 0.6475 | 0.000$ | 0.7711 |
| SA | 0.1571 | 0.000$ | 0.6635 | 0.000$ | 0.8206 | 0.2149 | 0.000$ | 0.5872 | 0.000$ | 0.8020 | 0.1610 | 0.000$ | 0.6296 | 0.000$ | 0.7906 | 0.2325 | 0.000$ | 0.5824 | 0.000$ | 0.8149 |
| $\boldsymbol{d_3}$ | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.1975 | 0.000$ | 0.7082 | 0.000$ | 0.9058 | 0.2820 | 0.000$ | 0.5918 | 0.000$ | 0.8738 | 0.2768 | 0.000$ | 0.5502 | 0.000$ | 0.8270 | 0.2383 | 0.000$ | 0.6301 | 0.000$ | 0.8684 |
| US | 0.0000 | 0.999 | 0.9338 | 0.000$ | 0.9338 | 0.0810 | 0.000$ | 0.8114 | 0.000$ | 0.8924 | 0.1701 | 0.000$ | 0.7148 | 0.000$ | 0.8848 | 0.1344 | 0.000$ | 0.8277 | 0.000$ | 0.9621 |
| Germany | 0.1313 | 0.000$ | 0.7354 | 0.000$ | 0.8667 | 0.1339 | 0.000$ | 0.7110 | 0.000$ | 0.8449 | 0.0959 | 0.000$ | 0.8097 | 0.000$ | 0.9055 | 0.1845 | 0.000$ | 0.6963 | 0.000$ | 0.8807 |
| UK | 0.3585 | 0.000$ | 0.5067 | 0.000$ | 0.8652 | 0.2254 | 0.000$ | 0.6238 | 0.000$ | 0.8492 | 0.5503 | 0.000$ | 0.0000 | 0.999 | 0.5503 | 0.0472 | 0.356 | 0.9220 | 0.000$ | 0.9693 |
| Italy | 0.1772 | 0.000$ | 0.6617 | 0.000$ | 0.8389 | 0.2377 | 0.000$ | 0.6489 | 0.000$ | 0.8865 | 0.1171 | 0.000$ | 0.7775 | 0.000$ | 0.8946 | 0.1127 | 0.000$ | 0.7782 | 0.000$ | 0.8909 |
| France | 0.1398 | 0.000$ | 0.7578 | 0.000$ | 0.8975 | 0.1695 | 0.000$ | 0.7109 | 0.000$ | 0.8804 | 0.3466 | 0.000$ | 0.4658 | 0.000$ | 0.8123 | 0.0927 | 0.000$ | 0.8587 | 0.000$ | 0.9514 |
| Canada | 0.2500 | 0.000$ | 0.5491 | 0.000$ | 0.7991 | 0.2488 | 0.000$ | 0.6473 | 0.000$ | 0.8961 | 0.3776 | 0.000$ | 0.5065 | 0.000$ | 0.8841 | 0.4625 | 0.000$ | 0.4269 | 0.000$ | 0.8894 |
| Brazil | 0.2204 | 0.000$ | 0.6587 | 0.000$ | 0.8791 | 0.2001 | 0.000$ | 0.7126 | 0.000$ | 0.9127 | 0.2519 | 0.000$ | 0.5966 | 0.000$ | 0.8484 | 0.1401 | 0.000$ | 0.7593 | 0.000$ | 0.8994 |
| Russia | 0.1035 | 0.000$ | 0.8031 | 0.000$ | 0.9066 | 0.0604 | 0.054* | 0.9218 | 0.000$ | 0.9821 | 0.1989 | 0.000$ | 0.7286 | 0.000$ | 0.9275 | 0.0979 | 0.000$ | 0.8321 | 0.000$ | 0.9300 |
| India | 0.1564 | 0.000$ | 0.7032 | 0.000$ | 0.8595 | 0.3098 | 0.000$ | 0.5702 | 0.000$ | 0.8800 | 0.1020 | 0.000$ | 0.7684 | 0.000$ | 0.8704 | 0.2034 | 0.000$ | 0.7031 | 0.000$ | 0.9065 |
| China | 0.2430 | 0.000$ | 0.6393 | 0.000$ | 0.8823 | 0.1298 | 0.000$ | 0.7793 | 0.000$ | 0.9091 | 0.0784 | 0.000$ | 0.7923 | 0.000$ | 0.8707 | 0.1790 | 0.820 | 0.0000 | 0.999 | 0.1790 |
| SA | 0.0575 | 0.000$ | 0.8168 | 0.000$ | 0.8743 | 0.4709 | 0.000$ | 0.2933 | 0.085* | 0.7642 | 0.1669 | 0.000$ | 0.7546 | 0.000$ | 0.9215 | 0.1533 | 0.000$ | 0.7155 | 0.000$ | 0.8688 |
| $\boldsymbol{d_4}$ | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.7194 | 0.000$ | 0.0000 | 0.999 | 0.7194 | 0.1123 | 0.000$ | 0.7615 | 0.000$ | 0.8738 | 0.2461 | 0.000$ | 0.6197 | 0.000$ | 0.8658 | 0.3234 | 0.000$ | 0.2661 | 0.000$ | 0.5895 |
| US | 0.1339 | 0.000$ | 0.7110 | 0.000$ | 0.8449 | 0.1115 | 0.000$ | 0.8215 | 0.000$ | 0.9330 | 0.1790 | 0.820 | 0.0000 | 0.999 | 0.1790 | 0.2176 | 0.000$ | 0.6463 | 0.000$ | 0.8639 |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Germany | 0.2248 | 0.000$ | 0.7226 | 0.000$ | 0.9475 | 0.4631 | 0.000$ | 0.2693 | 0.000$ | 0.7324 | 0.5767 | 0.000$ | 0.0041 | 0.953 | 0.5808 | 0.3592 | 0.002$ | 0.4230 | 0.148 | 0.7822 |
| UK | 0.1364 | 0.000$ | 0.8167 | 0.000$ | 0.9532 | 0.0380 | 0.002$ | 0.8590 | 0.000$ | 0.8970 | 0.2228 | 0.000$ | 0.7464 | 0.000$ | 0.9692 | 0.5264 | 0.000$ | 0.1130 | 0.120 | 0.6394 |
| Italy | 0.3551 | 0.000$ | 0.4076 | 0.000$ | 0.7628 | 0.7445 | 0.000$ | 0.0000 | 0.999 | 0.7445 | 0.4221 | 0.000$ | 0.2922 | 0.000$ | 0.7143 | 0.7652 | 0.000$ | 0.0000 | 0.999 | 0.7652 |
| France | 0.5094 | 0.000$ | 0.1798 | 0.007$ | 0.6893 | 0.7045 | 0.000$ | 0.0000 | 0.999 | 0.7045 | 0.1807 | 0.000$ | 0.7440 | 0.000$ | 0.9247 | 0.4598 | 0.000$ | 0.3400 | 0.001$ | 0.7998 |
| Canada | 0.3925 | 0.000$ | 0.3947 | 0.146 | 0.7872 | 0.4237 | 0.000$ | 0.1600 | 0.000$ | 0.5837 | 0.7121 | 0.000$ | 0.0726 | 0.275 | 0.7847 | 0.4234 | 0.000$ | 0.2956 | 0.000$ | 0.7190 |
| Brazil | 0.1039 | 0.000$ | 0.8356 | 0.000$ | 0.9396 | 0.7088 | 0.000$ | 0.0000 | 0.999 | 0.7088 | 0.1602 | 0.000$ | 0.7470 | 0.000$ | 0.9072 | 0.2542 | 0.000$ | 0.6591 | 0.000$ | 0.9133 |
| Russia | 0.5389 | 0.000$ | 0.0000 | 0.999 | 0.5389 | 0.1374 | 0.000$ | 0.8117 | 0.000$ | 0.9491 | 0.7893 | 0.000$ | 0.0586 | 0.110 | 0.8479 | 0.6938 | 0.000$ | 0.0000 | 0.999 | 0.6938 |
| India | 0.5908 | 0.000$ | 0.0000 | 0.999 | 0.5908 | 0.1814 | 0.000$ | 0.7529 | 0.000$ | 0.9344 | 0.6246 | 0.000$ | 0.1149 | 0.300 | 0.7395 | 0.2024 | 0.000$ | 0.7169 | 0.000$ | 0.9193 |
| China | 0.3207 | 0.000$ | 0.3729 | 0.002$ | 0.6935 | 0.3769 | 0.000$ | 0.2371 | 0.001$ | 0.6139 | 0.7451 | 0.000$ | 0.0532 | 0.388 | 0.7984 | 0.6422 | 0.000$ | 0.0168 | 0.740 | 0.6590 |
| SA | 0.4898 | 0.000$ | 0.0663 | 0.223 | 0.5562 | 0.2694 | 0.000$ | 0.3555 | 0.000$ | 0.6250 | 0.1512 | 0.013# | 0.6117 | 0.000$ | 0.7629 | 0.2138 | 0.000$ | 0.4956 | 0.015# | 0.7094 |
| $d_5$ | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.2894 | 0.000$ | 0.6962 | 0.000$ | 0.9856 | 0.4912 | 0.000$ | 0.4202 | 0.000$ | 0.9114 | 0.7270 | 0.022# | 0.0000 | 0.999 | 0.7270 | 0.4418 | 0.000$ | 0.5117 | 0.000$ | 0.9535 |
| US | 0.3648 | 0.000$ | 0.5637 | 0.000$ | 0.9285 | 0.3730 | 0.000$ | 0.5879 | 0.000$ | 0.9609 | 0.5662 | 0.000$ | 0.3256 | 0.000$ | 0.8918 | 0.8954 | 0.9876 | 0.4562 | 0.4366 | 0.0000 |
| Germany | 0.2994 | 0.000$ | 0.6417 | 0.000$ | 0.9411 | 0.4835 | 0.000$ | 0.3538 | 0.000$ | 0.8373 | 0.7890 | 0.000$ | 0.0281 | 0.672 | 0.8171 | 0.5496 | 0.000$ | 0.3572 | 0.000$ | 0.9068 |
| UK | 0.2355 | 0.000$ | 0.7504 | 0.000$ | 0.9859 | 0.7329 | 0.000$ | 0.0000 | 0.999 | 0.7329 | 0.7341 | 0.000$ | 0.0211 | 0.817 | 0.7552 | 0.7553 | 0.000$ | 0.0000 | 0.999 | 0.7553 |
| Italy | 0.3721 | 0.000$ | 0.5930 | 0.000$ | 0.9651 | 0.7965 | 0.000$ | 0.0000 | 0.999 | 0.7965 | 0.6194 | 0.000$ | 0.2490 | 0.000$ | 0.8684 | 0.5703 | 0.000$ | 0.1794 | 0.014# | 0.7497 |
| France | 0.6151 | 0.000$ | 0.2613 | 0.198 | 0.8763 | 0.4118 | 0.000$ | 0.2700 | 0.000$ | 0.6818 | 0.8060 | 0.000$ | 0.0000 | 0.999 | 0.8060 | 0.8148 | 0.000$ | 0.0000 | 0.999 | 0.8148 |
| Canada | 0.7555 | 0.000$ | 0.1359 | 0.019# | 0.8915 | 0.6630 | 0.000$ | 0.1875 | 0.000$ | 0.8505 | 0.6051 | 0.000$ | 0.2886 | 0.000$ | 0.8938 | 0.9112 | 0.6083 | 0.3765 | 0.9843 | 0.8245 |
| Brazil | 0.4922 | 0.000$ | 0.4343 | 0.000$ | 0.9266 | 0.3758 | 0.000$ | 0.5975 | 0.000$ | 0.9733 | 0.7759 | 0.000$ | 0.0000 | 0.999 | 0.7759 | 0.5461 | 0.000$ | 0.3160 | 0.000$ | 0.8621 |
| Russia | 0.2679 | 0.000$ | 0.5779 | 0.000$ | 0.8458 | 0.5828 | 0.000$ | 0.2122 | 0.000$ | 0.7949 | 0.4456 | 0.000$ | 0.5023 | 0.000$ | 0.9479 | 0.8193 | 0.000$ | 0.0000 | 0.999 | 0.8193 |
| India | 0.4113 | 0.000$ | 0.5392 | 0.000$ | 0.9504 | 0.7492 | 0.000$ | 0.0665 | 0.183 | 0.8157 | 0.5175 | 0.000$ | 0.4003 | 0.000$ | 0.9178 | 0.7509 | 0.000$ | 0.0852 | 0.420 | 0.8360 |
| China | 0.3361 | 0.000$ | 0.6228 | 0.000$ | 0.9589 | 0.3135 | 0.000$ | 0.5797 | 0.000$ | 0.8932 | 0.3012 | 0.000$ | 0.6695 | 0.000$ | 0.9708 | 0.1860 | 0.000$ | 0.7014 | 0.000$ | 0.8875 |
| SA | 0.5241 | 0.000$ | 0.2979 | 0.000$ | 0.8220 | 0.4470 | 0.000$ | 0.3011 | 0.000$ | 0.7480 | 0.5439 | 0.000$ | 0.3349 | 0.000$ | 0.8788 | 0.3183 | 0.000$ | 0.5529 | 0.000 | 0.8712 |
| $d_6$ | | | | | | | | | | | | | | | | | | | | |
| Japan | 0.2606 | 0.000$ | 0.7319 | 0.000$ | 0.9925 | 0.7708 | 0.000$ | 0.0000 | 0.999 | 0.7708 | 0.4987 | 0.000$ | 0.4388 | 0.000$ | 0.9375 | 0.6882 | 0.001$ | 0.2634 | 0.258 | 0.9516 |
| US | 0.5052 | 0.000$ | 0.3946 | 0.000$ | 0.8998 | 0.8316 | 0.000$ | 0.0000 | 0.999 | 0.8316 | 0.1548 | 0.000$ | 0.7749 | 0.000$ | 0.9297 | 0.8442 | 0.000$ | 0.0000 | 0.999 | 0.8442 |
| Germany | 0.5415 | 0.000$ | 0.3401 | 0.000$ | 0.8816 | 0.8882 | 0.000$ | 0.0000 | 0.999 | 0.8882 | 0.5989 | 0.000$ | 0.3222 | 0.009$ | 0.9211 | 0.3958 | 0.115 | 0.5734 | 0.045# | 0.9692 |
| UK | 0.0353 | 0.000$ | 0.9235 | 0.000$ | 0.9588 | 0.6078 | 0.000$ | 0.2323 | 0.000$ | 0.8401 | 0.4842 | 0.000$ | 0.3087 | 0.000$ | 0.7929 | 0.6030 | 0.000$ | 0.3285 | 0.000$ | 0.9315 |
| Italy | 0.3596 | 0.000$ | 0.6080 | 0.000$ | 0.9676 | 0.2957 | 0.000$ | 0.6826 | 0.000$ | 0.9783 | 0.1844 | 0.000$ | 0.7701 | 0.000$ | 0.9546 | 0.4626 | 0.000$ | 0.4239 | 0.000$ | 0.8865 |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| France | 0.5447 | 0.000$ | 0.2503 | 0.000$ | 0.7950 | 0.7479 | 0.000$ | 0.1419 | 0.000$ | 0.8898 | 0.4456 | 0.000$ | 0.5198 | 0.000$ | 0.9654 | 0.4393 | 0.000$ | 0.4129 | 0.000$ | 0.8522 |
| Canada | 0.3090 | 0.000$ | 0.6614 | 0.000$ | 0.9703 | 0.8153 | 0.000$ | 0.0635 | 0.190 | 0.8788 | 0.7293 | 0.000$ | 0.2030 | 0.174 | 0.9324 | 0.5016 | 0.000$ | 0.4630 | 0.000$ | 0.9647 |
| Brazil | 0.3732 | 0.000$ | 0.5879 | 0.000$ | 0.9611 | 0.5181 | 0.000$ | 0.3790 | 0.005$ | 0.8971 | 0.3055 | 0.000$ | 0.4674 | 0.000$ | 0.7729 | 0.4871 | 0.000$ | 0.4792 | 0.000$ | 0.9663 |
| Russia | 0.2390 | 0.000$ | 0.7206 | 0.000$ | 0.9595 | 0.2678 | 0.000$ | 0.6971 | 0.000$ | 0.9649 | 0.3050 | 0.000$ | 0.6499 | 0.000$ | 0.9549 | 0.6074 | 0.000$ | 0.2369 | 0.001$ | 0.8444 |
| India | 0.7263 | 0.000$ | 0.0000 | 0.999 | 0.7263 | 0.6466 | 0.000$ | 0.2644 | 0.000$ | 0.9110 | 0.4683 | 0.000$ | 0.4909 | 0.000$ | 0.9592 | 0.3071 | 0.000$ | 0.6729 | 0.000$ | 0.9801 |
| China | 0.3573 | 0.000$ | 0.5981 | 0.000$ | 0.9554 | 0.3631 | 0.000$ | 0.6100 | 0.000$ | 0.9731 | 0.1785 | 0.000$ | 0.8061 | 0.000$ | 0.9847 | 0.6505 | 0.000$ | 0.3108 | 0.000$ | 0.9612 |
| SA | 0.6512 | 0.000$ | 0.1687 | 0.108 | 0.8199 | 0.1976 | 0.000$ | 0.7982 | 0.000$ | 0.9958 | 0.3635 | 0.000$ | 0.6183 | 0.000$ | 0.9818 | 0.6681 | 0.000$ | 0.2563 | 0.030# | 0.9244 |

Note: $= Significance at the 0.01 level

#= Significance at the 0.05 level

*= Significance at the 0.10 level

Table 3 reports the pairwise DCC results for return series and transformed data. R represents the return series at level (non-decomposed). $d_1$, $d_2$, $d_3$, $d_4$, $d_5$, $d_6$ represents different time-frequencies as defined in Table 2.

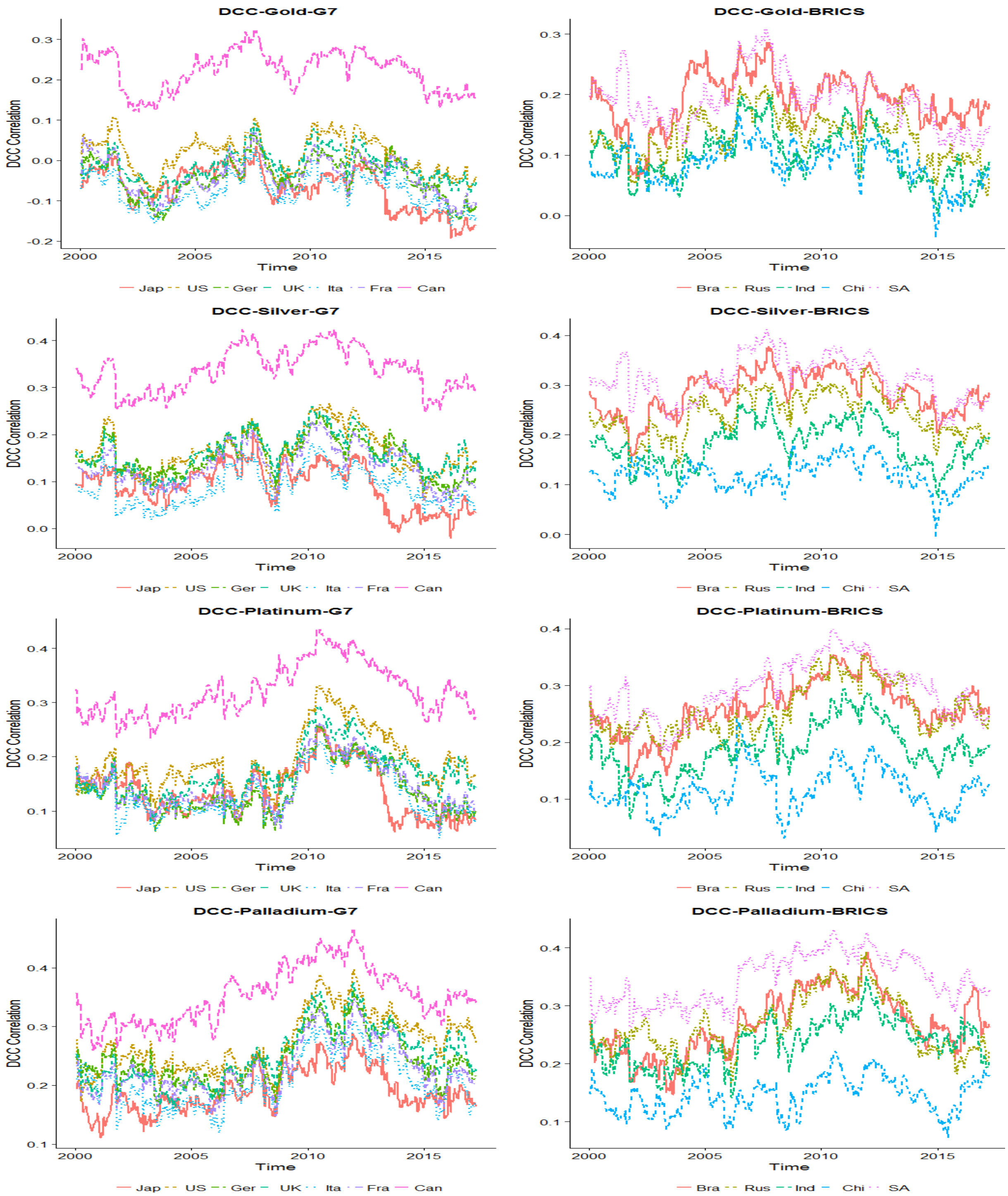



**Figure 4.** Dynamic conditional correlations between stock indices and precious metals at R (non-decomposed series)

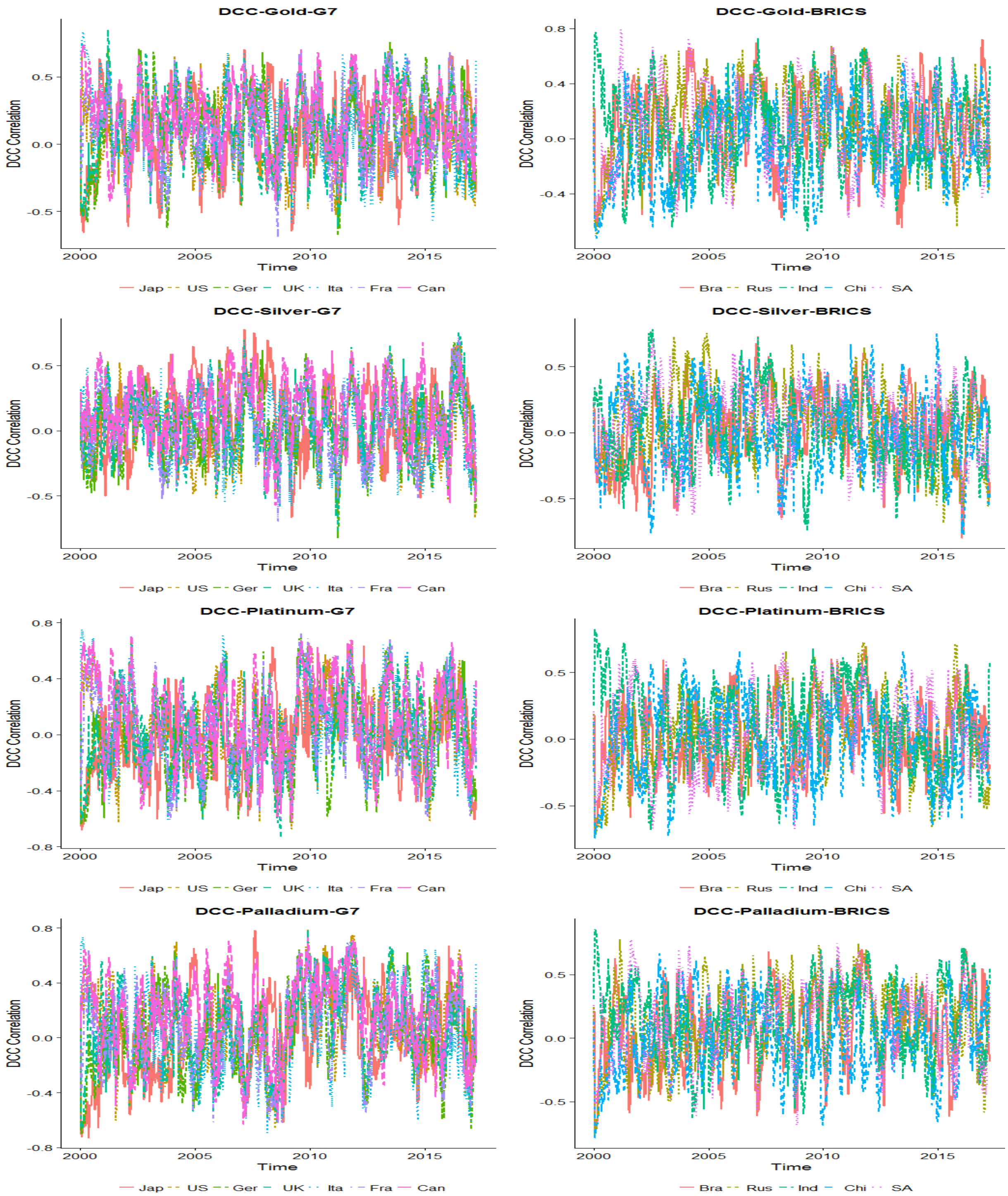



**Figure 5.** Dynamic conditional correlations between stock indices and precious metals at $d_3$ (decomposed series). Refer Table 2 for details.

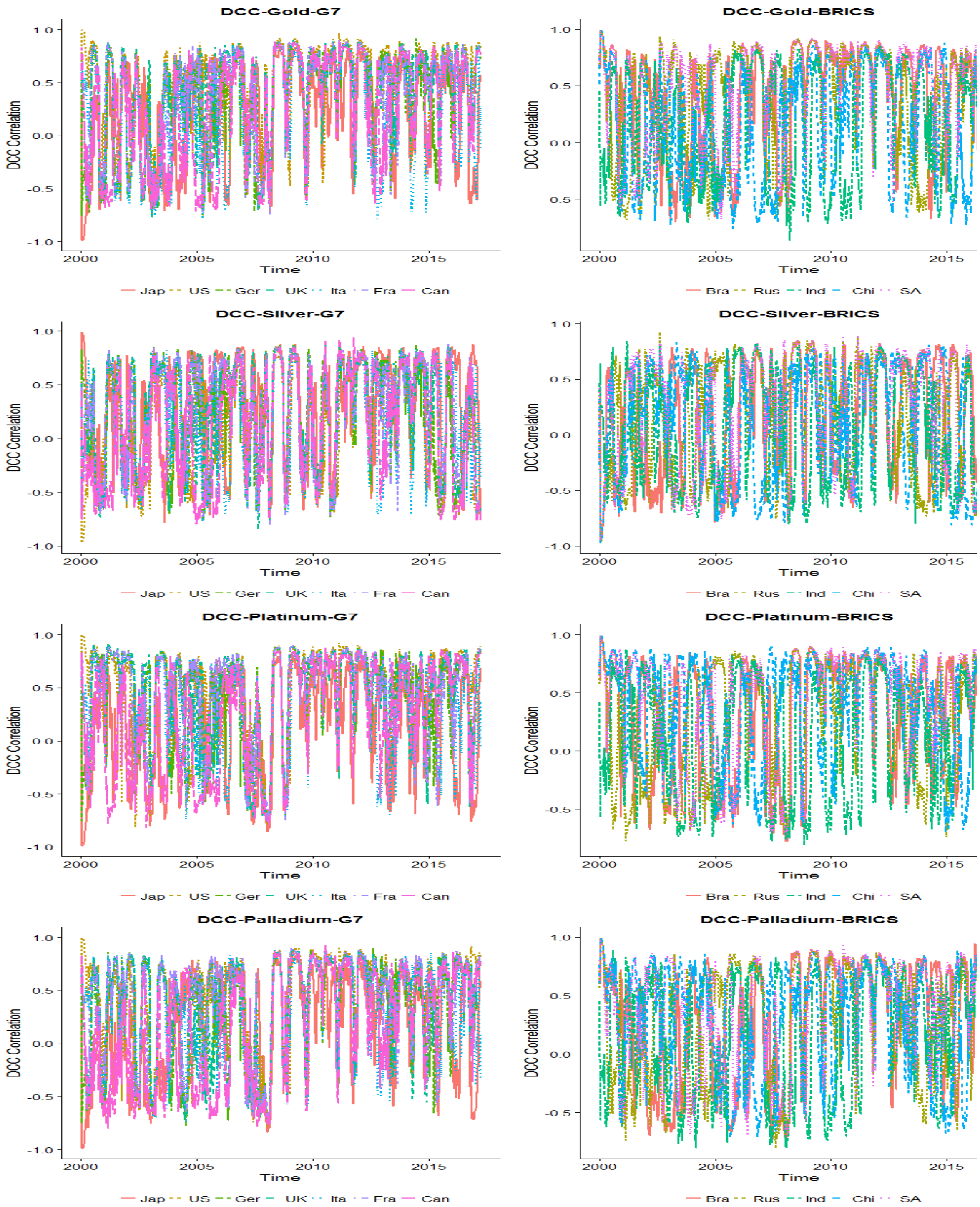
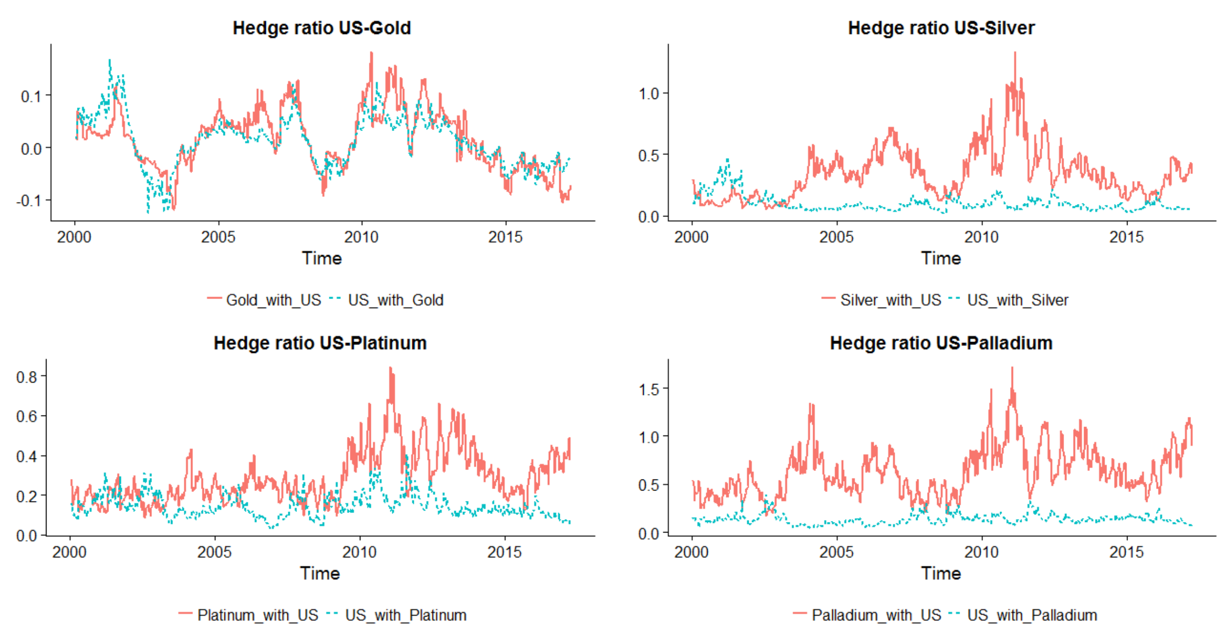



**Figure 6.** Dynamic conditional correlations between stock indices and precious metals at $d_6$ (decomposed series). Refer Table 2 for details.

**Table 4.** Hedging ratios for stock indices of US and India with precious metals

| | R | | | | $d_3$ | | | | $d_6$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Sd | Min | Max | Mean | Sd | Min | Max | Mean | Sd | Min | Max |
| Gold with US | 0.02 | 0.06 | -0.12 | 0.18 | 0.13 | 0.37 | -0.85 | 2.11 | 0.59 | 0.79 | -1.40 | 4.89 |
| Silver with US | 0.34 | 0.21 | 0.05 | 1.33 | 0.08 | 0.62 | -2.03 | 2.34 | 0.46 | 1.38 | -4.91 | 5.85 |
| Platinum with US | 0.29 | 0.13 | 0.09 | 0.84 | 0.12 | 0.47 | -1.99 | 1.98 | 0.70 | 0.83 | -1.51 | 3.87 |
| Palladium with US | 0.63 | 0.25 | 0.17 | 1.72 | 0.22 | 0.74 | -2.42 | 2.48 | 0.94 | 1.41 | -3.61 | 7.46 |
| US with Gold | 0.01 | 0.05 | -0.13 | 0.17 | 0.10 | 0.29 | -0.96 | 1.51 | 0.64 | 0.83 | -2.70 | 5.32 |
| US with Silver | 0.10 | 0.07 | 0.02 | 0.48 | 0.03 | 0.18 | -0.47 | 1.11 | -0.05 | 1.24 | -28.73 | 1.91 |
| US with Platinum | 0.14 | 0.06 | 0.03 | 0.41 | 0.04 | 0.24 | -1.24 | 0.94 | 0.45 | 0.54 | -1.91 | 2.17 |
| US with Palladium | 0.13 | 0.05 | 0.04 | 0.39 | 0.05 | 0.16 | -1.04 | 0.51 | 0.28 | 0.44 | -1.17 | 6.07 |
| Gold with India | 0.08 | 0.05 | 0.00 | 0.31 | 0.05 | 0.24 | -0.83 | 0.88 | 0.13 | 1.59 | -44.63 | 1.86 |
| Silver with India | 0.26 | 0.13 | 0.05 | 0.75 | 0.05 | 0.42 | -1.48 | 2.01 | 0.03 | 1.13 | -3.33 | 9.11 |
| Platinum with India | 0.20 | 0.08 | 0.05 | 0.46 | 0.07 | 0.34 | -1.45 | 1.41 | -0.03 | 4.99 | -147.85 | 2.29 |
| Palladium with India | 0.38 | 0.12 | 0.15 | 0.92 | 0.24 | 0.47 | -1.01 | 1.73 | 0.29 | 1.71 | -39.00 | 3.28 |
| India with Gold | 0.13 | 0.07 | 0.00 | 0.41 | 0.10 | 0.51 | -1.91 | 2.06 | 0.22 | 0.77 | -2.69 | 2.65 |
| India with Silver | 0.15 | 0.08 | 0.04 | 0.59 | 0.01 | 0.30 | -1.74 | 1.33 | 0.01 | 0.51 | -2.15 | 2.84 |
| India with Platinum | 0.19 | 0.07 | 0.08 | 0.45 | 0.13 | 0.40 | -0.99 | 1.84 | 0.15 | 0.58 | -1.43 | 1.74 |
| India with Palladium | 0.16 | 0.05 | 0.06 | 0.42 | 0.10 | 0.23 | -0.91 | 0.79 | 0.13 | 0.37 | -1.12 | 0.98 |



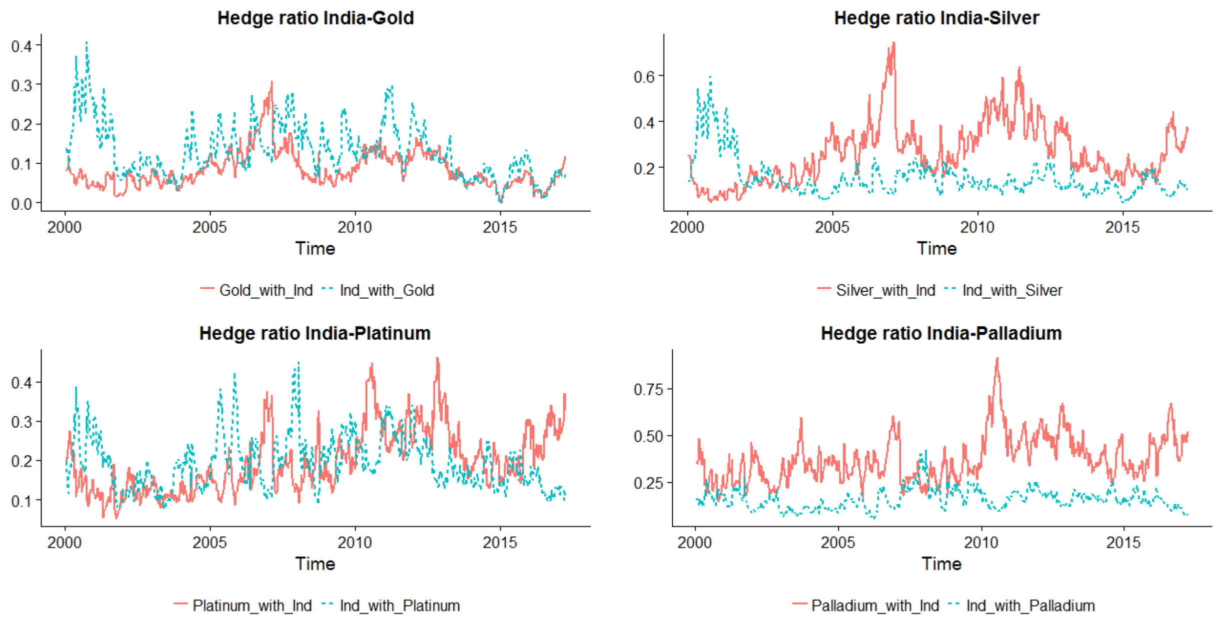
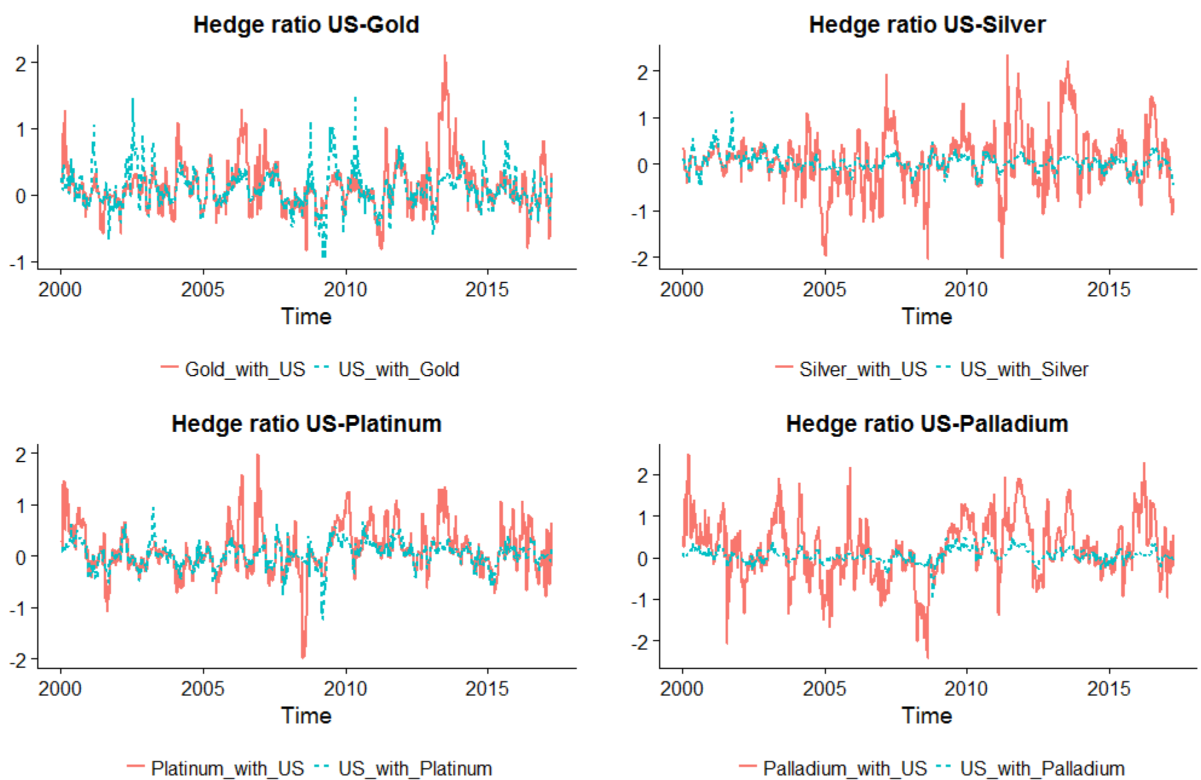



**Figure 7.** Hedge ratios between stock indices of US and India with precious metals at R



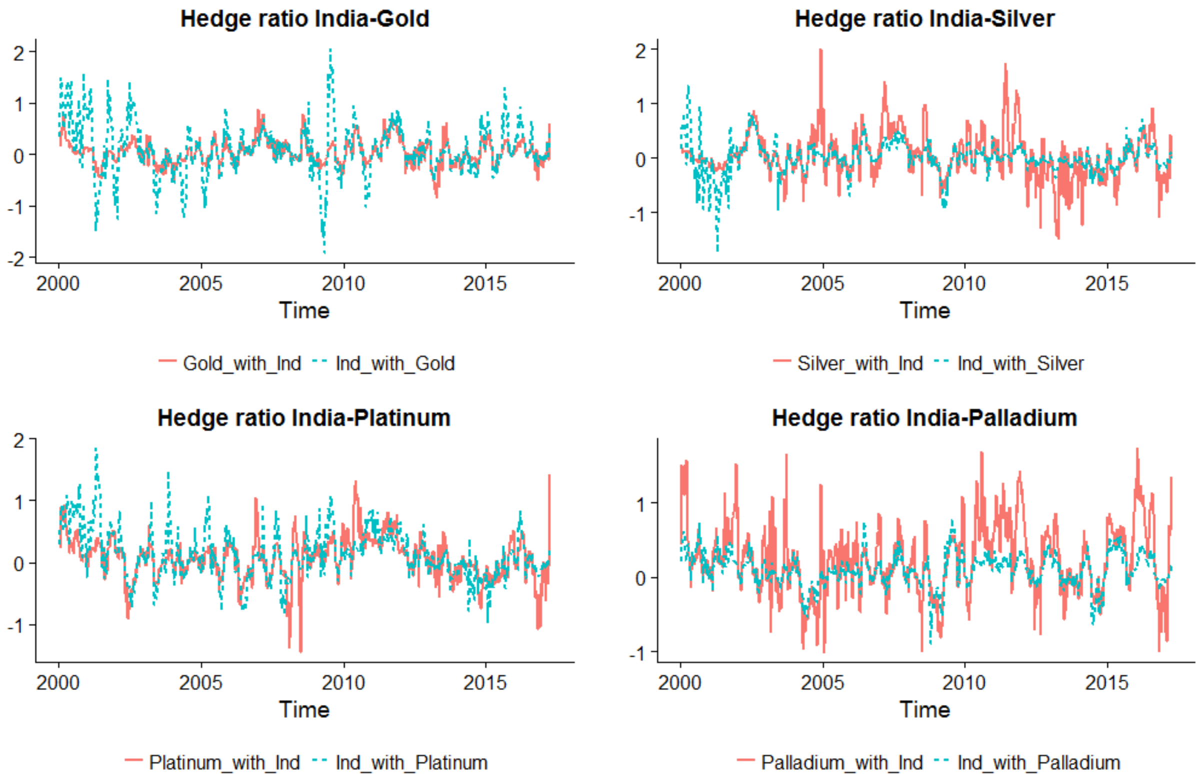



**Figure 8.** Hedge ratios between stock indices of US and India with precious metals at $d_3$

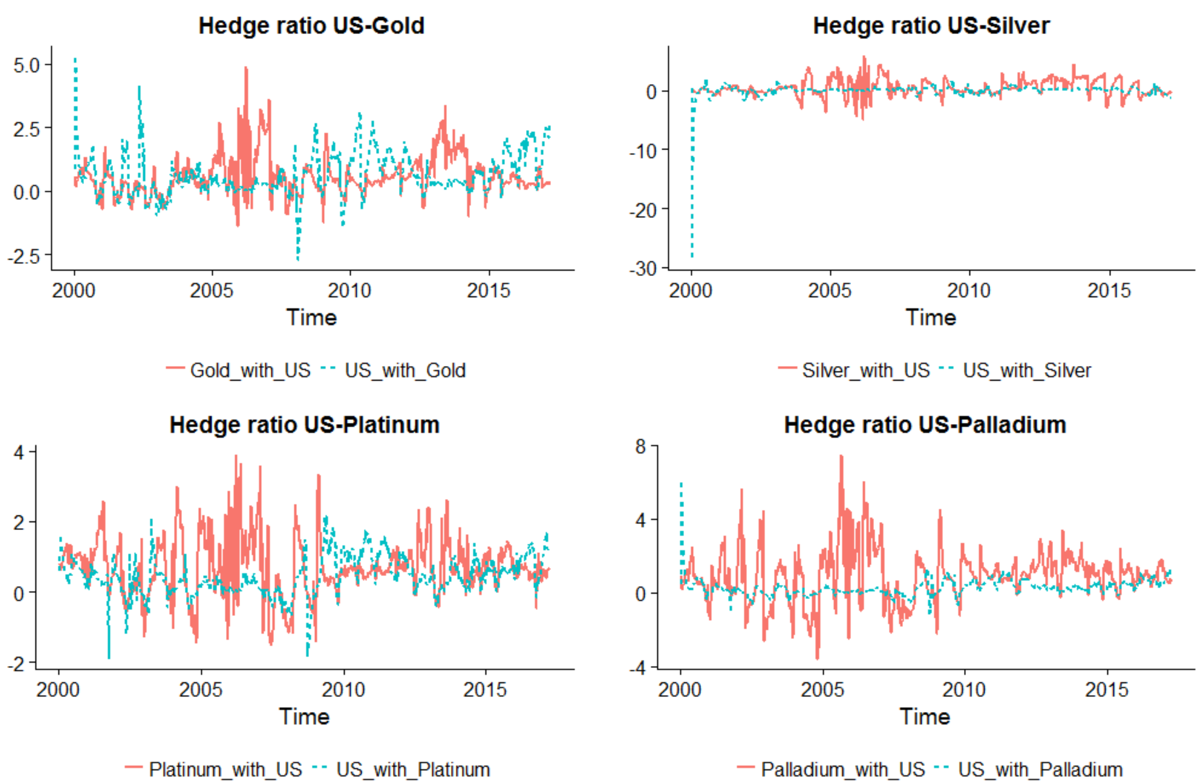



**Figure 9.** Hedge ratios between stock indices of US and India with precious metals at $d_6$

**Table 5.** Portfolio weights for stock indices of US and India with precious metals

| | R | | | | $d_3$ | | | | $d_6$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mean | Sd | Min | Max | Mean | Sd | Min | Max | Mean | Sd | Min | Max |
| Gold/US | 0.44 | 0.16 | 0.12 | 0.85 | 0.46 | 0.25 | -0.23 | 1.11 | 0.56 | 0.57 | -0.73 | 1.97 |
| Silver/Us | 0.24 | 0.22 | 0 | 0.89 | 0.26 | 0.21 | -0.18 | 1.03 | 0.27 | 0.37 | -0.38 | 1.23 |
| Platinum/US | 0.32 | 0.17 | 0.02 | 0.83 | 0.36 | 0.22 | -0.12 | 0.98 | 0.38 | 0.57 | -0.59 | 3.74 |
| Palladium/US | 0.12 | 0.13 | 0 | 0.76 | 0.16 | 0.17 | -0.26 | 0.72 | 0.11 | 0.38 | -0.77 | 2.42 |
| | | | | | | | | | | | | |
| Gold/India | 0.60 | 0.13 | 0.2 | 0.95 | 0.64 | 0.21 | 0.06 | 1.21 | 0.56 | 0.36 | -0.43 | 1.37 |
| Silver/India | 0.36 | 0.21 | 0.03 | 0.97 | 0.38 | 0.23 | -0.21 | 1.03 | 0.32 | 0.35 | -0.33 | 1.39 |
| Platinum/India | 0.50 | 0.16 | 0.15 | 0.89 | 0.55 | 0.24 | -0.1 | 1.23 | 0.43 | 0.35 | -0.32 | 1.39 |
| Palladium/India | 0.26 | 0.14 | 0.01 | 0.76 | 0.28 | 0.2 | -0.46 | 0.88 | 0.20 | 0.28 | -0.51 | 0.98 |

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