THEORY OF STRUCTURAL CHANGE IN DEVELOPING ECONOMIES

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Abstract

Economic growth is associated with (i)changes in sectoral output composition and (ii)changes in relative prices. Moreover, the recent US and India data, as a representative of developed and developing economies, respectively, show (iii)persistence of structural change in developing economies. This paper tries to develop a tractable growth-theoretic model of structural change for developing economies in a two-sector framework which is consistent with (i)-(iii), at the same time focus on the role of relative prices as an intertemporal equilibrating variable to maintain balanced growth which is not present in the literature. The paper shows that changes in the relative price path and structural change are endogenous properties of the model due to sectoral imbalances at the initial stages of development and the aggregate behavior is in line with Kaldor facts. The result also indicates that policies that attempt to rectify sectoral imbalances are as important as monetary policies that tackle inflation in the developing economies.

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2.1 INTRODUCTION

A well-documented feature of economic growth across countries is the associated changes in the composition of sectoral output, employment, and consumption structure (Kuznets, 1973; Maddison, 1980) known as ‘structural change.' Two of the stylized facts of structural change are the following (Boppart, 2014):

(i) Falling expenditure share of goods (against services),

(ii) Falling relative price of goods compared to services.

These facts suggest that as per capita income rises, expenditure shares and relative price turn towards income and price elastic goods. Are the facts (i) and (ii) mutually independent or related? Stated differently, is there a link between structural change and relative price movements in an economy? If such a link exists, it may offer insights on ‘inflation targeting’ in developing economies.

This paper argues that changes in relative price and structural change are an outcome of how imbalances in the sectoral growth rates of a developing economy are corrected over time. The closing of growth gap is accompanied by continuous changes in the market clearing relative price, leading to changes in output composition, and in turn giving rise to structural change. The dynamics of the relative price over time is a characteristic of the complete growth path of the economy.

The foundation of the literature on structural change models was led by the works of Kuznets (1957, 1973) and Kaldor (1961). Kuznet facts are defined by the changes in sectoral employment composition as the economy passes through different stages of development whereas Kaldor facts are defined as balanced growth of aggregate variables. Recently, the growth experience of several economies points to the simultaneous existence of Kaldor and Kuznets facts. These empirics ensued a growing interest in developing a multisector model which can simultaneously explain the Kaldor and Kuznets facts. The models can be classified into two categories, the demand side models, and the supply side models. The demand side models focus on the impact of income effect which leads to structural change through non-homothetic preferences, such as Kongsamut, Rebelo, and Xie (2001), Foelmmi and Zweimuller (2008), and Carerra and Raurich (2015). Kongsamut et al. (2001) assume a Stone-Geary preference while Foelmmi and Zweimuller (2008) assumes a hierarchy of preference to generate structural change in their models. Carerra and Raurich (2015) is an extension of Kongsamut, Rebelo, and Xie (2001). They have shown that slight deviation from the knife edge condition of Kongsamut et al. (2001) is consistent with balanced growth and structural change, but it requires a stringent restriction on the value of the initial level of aggregate income. The supple side models mainly focus on the substitution effect due to relative price changes to show structural change in their models. The noted papers in this area are Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). In Ngai and Pissarides (2007), there is exogenous total factor productivity difference across sectors which results in decrease in price in one sector and thus increasing demand, resulting in structural change whereas in Acemoglu and Guerrieri (2008), the structural change is due to factor proportion difference and capital deepening, and structural change is an asymptotic phenomenon.

The problem with demand side models is they exclude relative price effect to achieve balanced growth. Kongsamut et al. (2001), shows balanced growth in their model through a knife-edge condition which implies constant relative prices, and Foellmi and Zweimuller (2008), excludes price effect by assuming that technological difference is uncorrelated with hierarchical position of a good. The supply side models also have the same problem, in a way both Acemoglu and Guerrieri (2008) and Ngai and Pissarides (2007), abstract from non-homotheticity of preference to show the simultaneous existence of Kaldor and Kuznets facts. However, structural change is a result of both relative price effect and income effect. Buera and Kaboski (2009) and Boppart (2014) are the only two papers in this direction. Buera and Kaboski (2009), incorporates both relative price effect and income effect in an integrated framework with the demand side comprising of a nested CES function with minimum consumption requirements and the relative price effect through the supply side is characterized by exogenous total factor productivity difference across the sector. However, their model was unable to replicate the steep decline in manufacturing relative to service in US data, and they had to assume an unnaturally low elasticity of substitution of goods across the sector to fit the consumption and output data. Boppart (2014), also develops a theoretical model which integrates both relative price effect and income effect by formulating an indirect utility function which is a subclass of PIGL preferences. However, his model can replicate the structural change and relative price dynamics in the US data for agricultural and manufacturing goods, and it is also able to replicate the cross-section expenditure structure difference in the US data.

All the above models have tried to reconcile structural change and balanced growth simultaneously, but none of the models focus on the transition dynamics and the role of relative price as an intertemporal equilibrating variable to achieve balanced growth across the sectors. The theoretical contribution of this paper is to provide a model of structural change for developing economies, which exclusively focuses on the price path, its role as an intertemporal equilibrating variable to maintain balanced growth, and its related impact on inflation. The rest of the paper is organized as follows; section 2.2 provides a motivating example for the structural change theory, section 2.3 derives the theoretical model, section 2.4 shows the dynamics of relative prices, section

2.5 showcases the Kaldor and Kuznets fact, section 2.6 illustrates the impact of elasticity of substitution, and section 2.7 concludes. The next section gives a motivating example, why a theory of structural change is necessary for developing economies.

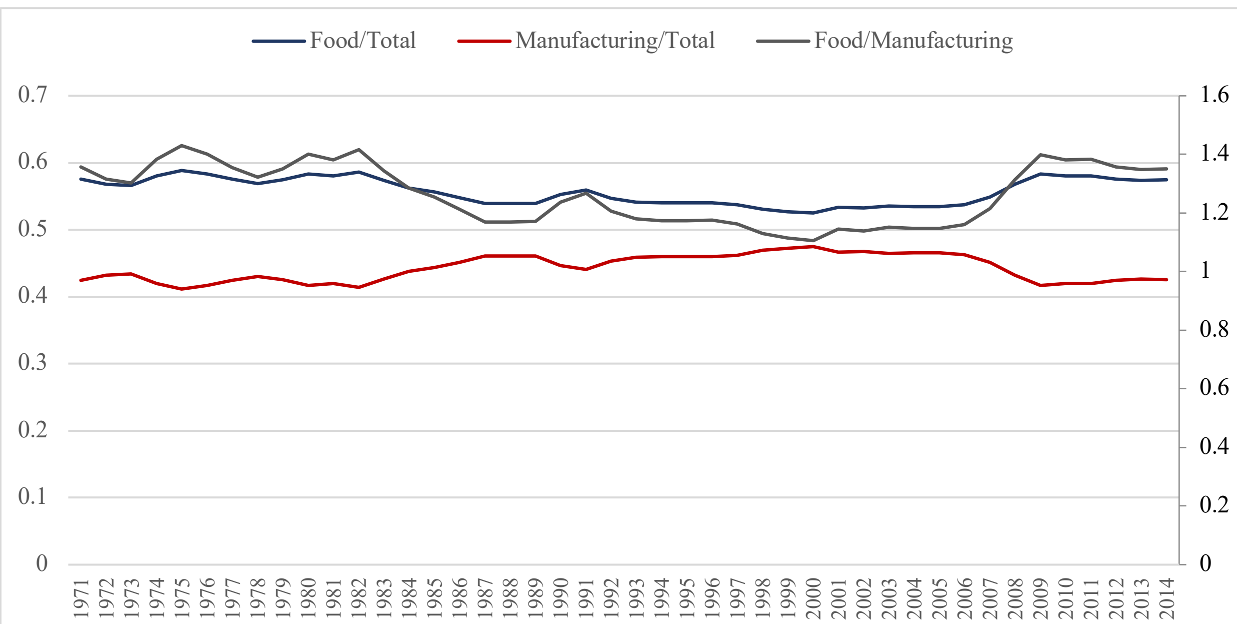
2.2 MOTIVATING EXAMPLE

I motivate this paper's theoretical model by comparing the agriculture (food)[3] and manufacturing sector data of a developed economy and a developing economy. I take the US and India data as representative of a developed and developing economy, respectively. Figure 1 plots the expenditure share of food and manufacturing as a fraction of total household personal consumption expenditure data for the US. Figure 2 plots the relative price of agriculture for the US data. Figure 3 and Figure 4 do the same, for the Indian dataset. Figure 1, shows both food and manufacturing has a downward trend for the US data with the gap between them closing during 2003 and thereafter increasing thus showing no structural change in the US data. The corresponding relative price of agriculture goods is shown in Figure 2. The logarithmic trend line for the US data in Figure 2 shows almost a linear trend, conforming to the conclusion of no structural change for the US data. The corresponding graphs for the Indian data are presented in Figure 3 and Figure 4, respectively. Unlike the US data, Figure 3 illustrates the manifestation of Engel Law in India. The increase in relative price of agricultural goods is shown in Figure 4.

[3] The agriculture data for US includes beverages also as the object wise data was not separable.

The increase in relative prices in Figure 4 creates a conundrum because if the elasticity of substitution between two sectors is less than unity[4], then an increase in the price of one sector will also increase its expenditure share. Moreover, the changes in expenditure structure (Figure 3), seems to be persisting, continuing for more than three decades. These two pieces of evidence call for a theoretical perspective which focuses exclusively on the out of steady state behavior of the relative price path and simultaneously, explains why structural change is a long-run phenomenon in a developing economy. In the subsequent section, I build the theoretical model, explaining the cause of the long-run structural change and the movement of the price path with a discussion about inflation targeting regime from the perspective of a developing economy, the asymptotic equilibrium and non-balanced growth and impact of elasticity of substitution between capital and labor.

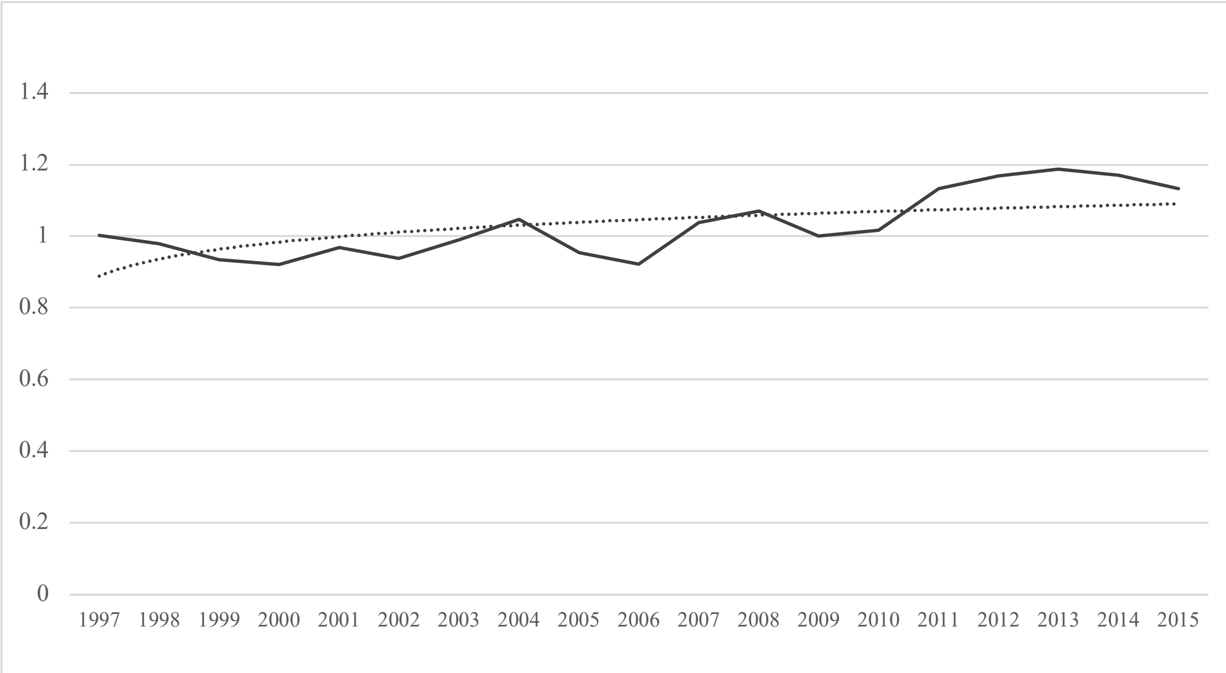
Figure ***Error! No text of specified style in document.****-1: Expenditure Share: US*



Source: BEA, NIPA table 2.4.5

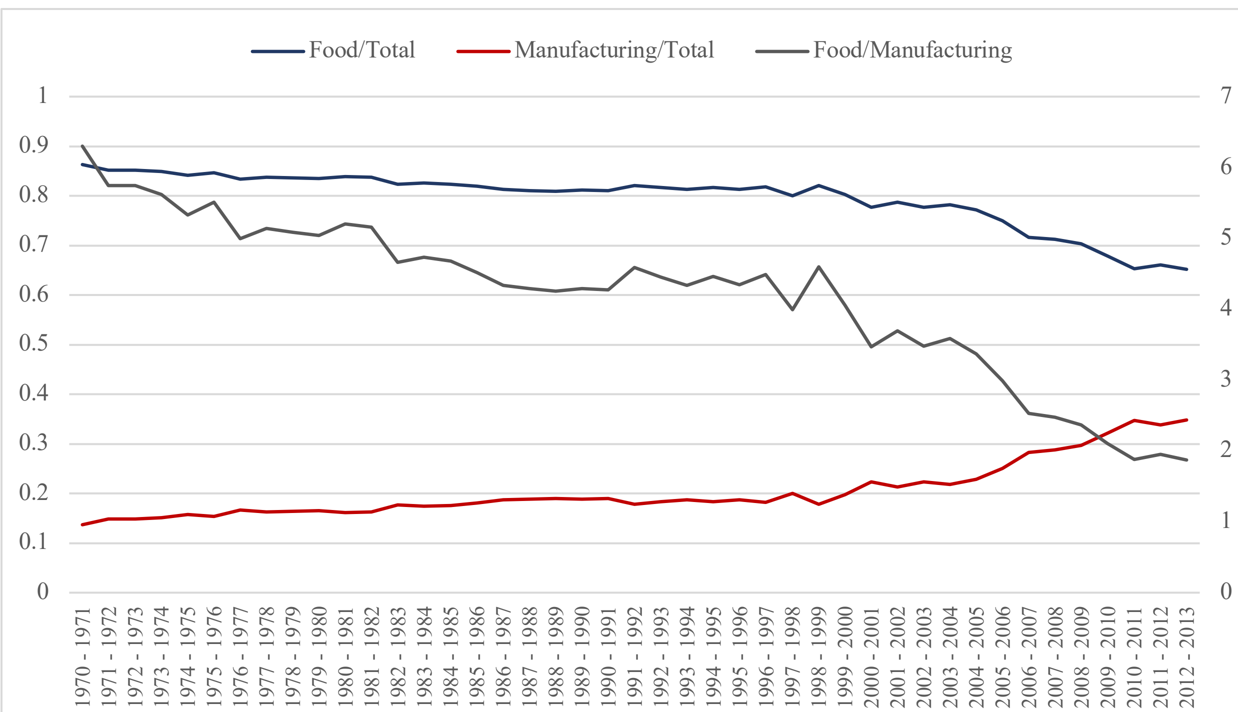
[4] This is considered as the empirically relevant case. For further clarification, see Acemoglu and Guerrieri (2008).

*Figure **Error! No text of specified style in document.**-2: Movement of Relative Price of Agriculture: US*



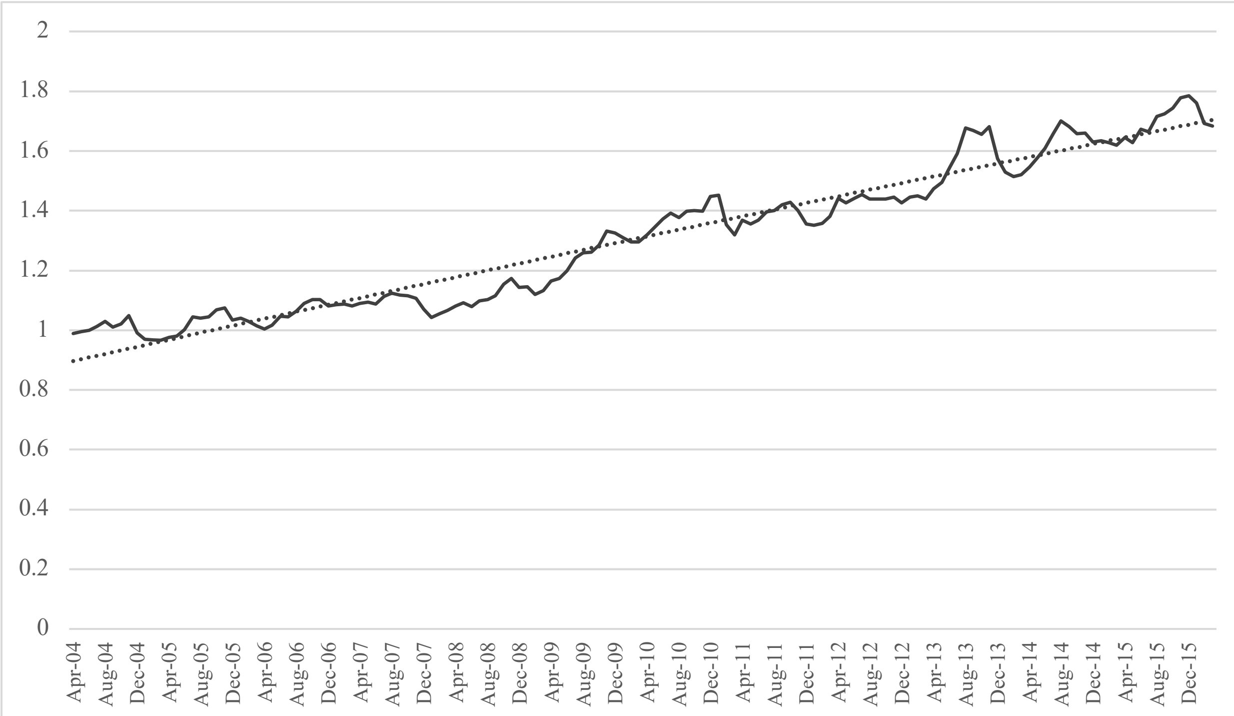
Source: BEA, NIPA table

*Figure **Error! No text of specified style in document.**-3: Expendіture Share: India*



Source: Adapted from "Inflation Targeting amidst Structural Change: Some Analytics for Developing Economies" by S. Chattopadhyay, 2017, *Economic and Political Weekly*, 52(2), p. 81.

Figure ***Error! No text of specified style in document.****-4: Movement of Relative Price of Agriculture: India*



Source: Adapted from "Inflation Targeting amidst Structural Change: Some Analytics for Developing Economies" by S. Chattopadhyay, 2017, *Economic and Political Weekly*, 52(2), p. 81.

2.3 THEORETICAL MODEL

I consider a closed economy with two sectors, agriculture (Y_A-sector) and manufacturing (Y_M-sector). The agricultural sector produces food with an expenditure elasticity of demand less than unity while the manufacturing sector produces clothing, footwear, furniture and other household appliances with an expenditure elasticity of demand strictly greater than unity. There is surplus labor in the agricultural sector. Agricultural sector produces food for self-consumption and the manufacturing sector. Let the food available for the manufacturing sector after netting out self-consumption is X_A, also known as 'marketed surplus.' This surplus is fixed in the short run.

Let the relative price of food be 'P' ($=P_A/P_M$). Marketed surplus, X_A, is consumed by the Y_M sector laborers. The proceed 'PX_A' goes to the landowners and big traders of the Y_A-sector and is used to buy capital goods from the Y_M-sector. This means that a rise in P (and hence in PX_A) does not cause this group to demand more food, thereby causing a fall in marketed surplus in the same period. If that happens, 'P' would increase further, might leading to an explosive path of 'P.' Such a possibility is ruled out by the assumption that demand for food by this group has already been satisfied, and there are no further income and relative price effects on the food demand.

2.3.1 Demand Side: The demand side of the economy is defined as in Weiss & Boppart (2013). The concept of marketed surplus leads us to redefine the concept of representative households in the model economy. I define households as, households who are not related to the process of agricultural production, i.e., I include only the manufacturing sector laborers in the definition of households. The households are endowed with 'L' units of inelastically supplied labor and 'A(0)' units of initial wealth. The households have an indirect utility function defined as

$$V(P,E)=\frac{1}{\varepsilon}E^{\varepsilon}-\frac{\beta}{\gamma}P^{\gamma}-\frac{1}{\varepsilon}+\frac{\beta}{\gamma} \tag{1}$$

Where $'E'$ denotes the nominal expenditure level, $'P'$ denotes the relative price of agriculture goods[5], $'\beta'$ determines the type/functional form of the preference, $'\varepsilon'$ determines the degree of non-homotheticity, and $'\gamma'$ determines the elasticity of substitution between the agriculture and the manufacturing good, $0\le\varepsilon\le\gamma<1$ and $\beta,\gamma>0$. The intratemporal utility function defined in equation (1), is a subclass of "price independent generalized linearity" (PIGL) defined by Muellbauer (1975) and Muellbauer (1976)[6]. The reason for using this particular subclass of PIGL preference, the particular functional form is jointly consistent in a two-sector framework, with a decreasing expenditure share in one sector and an increasing expenditure share

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[5] I have normalized the price of manufacturing goods to 1

[6] For proof, regarding the standard properties of the utility function and existence of a non-negative consumption, see Boppart (2014).

in the other sector. Moreover, it also helps us in avoiding an aggregation problem, that the expenditure shares of the aggregate economy in multi-sector framework coincides with those of a household with a "representative" expenditure level. Lastly, the PIGL preference avoids the discontinuity problem of Stone-Geary preference and at the same time provides us with a patently non-linear Engel curves as shown below in Figure 5 and Figure 6.

Intratemporal Optimization: Applying Roy's identity yields the following demand system[7]

$$X_A(t) = \beta E^{1-\varepsilon} P^{\gamma-1} \tag{2}$$

$$X_M(t) = E(t) - \beta E^{1-\varepsilon} P^{\gamma} \tag{3}$$

The expenditure share devoted to agricultural goods and manufacturing goods is shown in equation (4) and equation (5) respectively. The expenditure system reveals that Engel's law applies

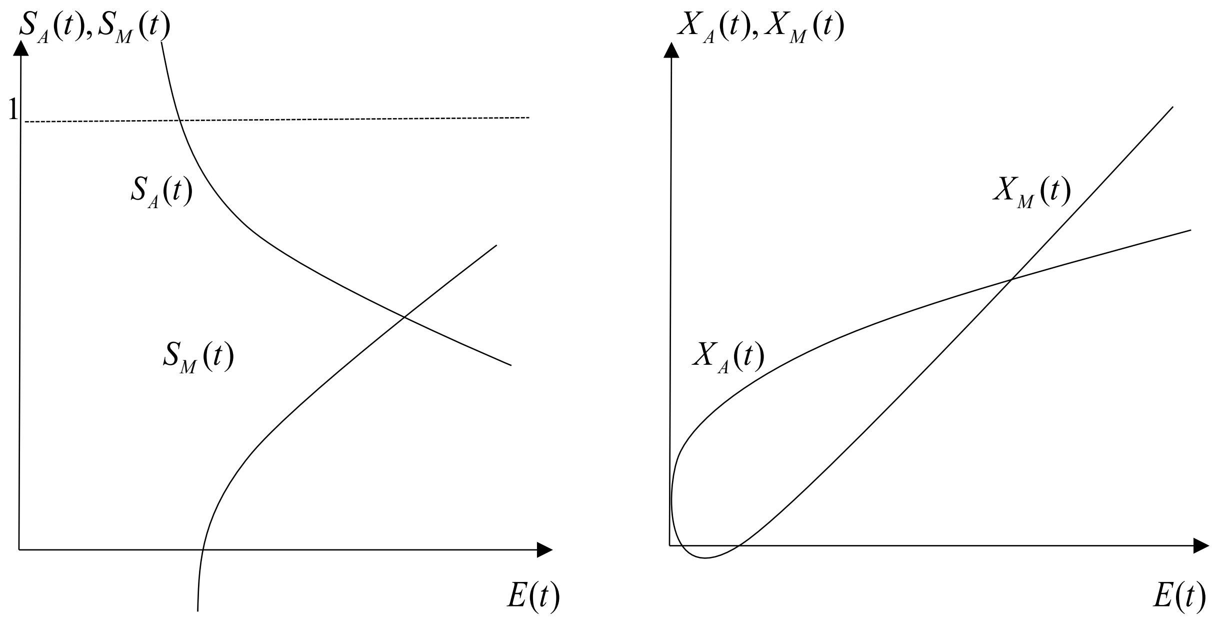
$$S_A(t) = \beta E^{-\varepsilon} P^{\gamma} \tag{4}$$

$$S_M(t) = 1 - \beta E^{-\varepsilon} P^{\gamma} \tag{5}$$

, thus implying non-homotheticity. For $\varepsilon > 0,$ Figure (5) and Figure (6), plots the Engel curve and sectoral expenditure share as functions of the expenditure level as shown in Boppart (2014). The non-linear Engel curve shows that preferences are non-homothetic in nature. The expenditure elasticity of agriculture goods is given by $1-\varepsilon$, and the elasticity of substitution is given by $1-\gamma+(\varepsilon-\gamma)\left(\dfrac{S_A}{1-S_A}\right)$. So, the elasticity of substitution can be either larger or smaller than unity depending upon the parameter γ. This means that both the income and substitution channel are present in the model, and it is controlled by the parameters γ and ε.

Figure II 5: Expenditure Shares. Adapted from "Structural Change and The Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences" by T. Boppart, 2014, *Econometrica,* 82(6), p. 2180.

Figure II 6: Engel Curves. Adapted from "Structural Change and The Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences" by T. Boppart, 2014, *Econometrica,* 82(6), p. 2181.



Intertemporal Optimization: The household takes ω (nominal manufacturing product wage) and r, as given and maximizes their intertemporal preference,

$$U_i(0) = \int_0^\infty \exp[-(\rho t)]V(P(t), E(t))dt$$

subject to the following flow budget constraint and the transversality condition,

$\dot{A}(t) = r(t)A(t) + \omega(t)L - E(t)$ and $\lim_{t\to\infty} E(t)^{\varepsilon-1} A(t)\exp[-\rho t] = 0$

the optimization yields the following result,

$$\frac{\dot{E}(t)}{E(t)} = \frac{1}{(1-\varepsilon)}\left[r(t) - \rho\right] \tag{6}$$

Proof: The current valued Hamiltonian reads

$$\mathrm{H} = V(E(t), P(t)) + \lambda(t)[r(t)A(t) + \omega(t)L - E(t)].$$

The first order conditions are

$$E(t)^{\varepsilon-1} - \lambda(t) = 0,$$
$$r(t)\lambda(t) = \rho\lambda(t) - \dot{\lambda}(t).$$

After doing some algebraic manipulations and simplifying, one will get equation (6). *Q.E.D.*

This is the same form of Euler equation as in Boppart (2013), but there is a subtle difference in the result I get and Boppart (2013). I have assumed that the real wage in terms of food is constant unlike Boppart (2013), which allows us an interesting dynamics in terms of the movement of nominal manufacturing product wage which I discuss in Section 2.5.1.

2.3.2 Production Side: There is two output goods sector: the output of the agricultural good and the output of the manufacturing good. The output of the manufacturing good is also used to produce investment good, i.e., $Y_M = C_M$ + Investment (Inv.), C_M (demand for durable consumption goods) and Inv. are perfect substitutes, and the relative price between CM and Inv. is unity. The agricultural sector output is produced by a decreasing returns to scale technology. As shown in equation (7) and the manufacturing sector output is produced using constant elasticity to scale technology (CES) as shown in equation (8).

$$Y_A = bK_A{}^{\kappa} \tag{7}$$

$$Y_M = [\alpha K_M{}^{\frac{\xi-1}{\xi}} + \delta L^{\frac{\xi-1}{\xi}}]^{\frac{\xi}{\xi-1}} \tag{8}$$

'Y_A', is the output (marketed surplus) of food, $0 < \kappa < 1$, imposes the decreasing returns to scale restriction on the production technology of agricultural sector. 'Y_M', is the output in the manufacturing sector, where, 'ξ' is the elasticity of substitution in between the factors of production, α and δ are defined as functions of $\frac{1-\xi}{\xi}$, i.e., $\alpha = g\left(\frac{1-\xi}{\xi}\right), \delta = h\left(\frac{1-\xi}{\xi}\right)$, such that

$\alpha + \delta = 1$ when $\frac{1-\xi}{\xi} = 0$, i.e., $\xi = 1$ (Cobb-Douglas production function). I impose this restriction so that the function is constant returns to scale for all values of ξ. L, K_A, K_M, denotes the labor supply in the manufacturing sector and capital stock in the agricultural and manufacturing sector, respectively. The source of L is the surplus labor in the Y_A-sector. I assume that C_M is demanded only by the manufacturing laborers, and Inv. is demanded by the capital owners of the Y_M-sector and the land owners of the Y_A-sector.

Time differentiating equation (7), I get,

$$\dot{Y}_A = \kappa b K_A^{\kappa-1}.\dot{K}_A = \kappa Y_A.\frac{\dot{K}_A}{K_A}$$

$$\Rightarrow \frac{\dot{Y}_A}{Y_A} = \kappa \frac{\dot{K}_A}{K_A}$$

as I am interested in the movement of P, I assume that the growth rate in marketed surplus of food is fixed at $\overline{x}$.

$$i.e. \frac{\dot{Y}_A}{Y_A} = \overline{x} \tag{9}$$

The product wage and real wage are intrinsically linked. A rise in the cost of living (of which food is a major component) will call for an increase in the nominal wage in the manufacturing sector to keep the real wage the same. Now the nominal wage equals output price multiplied by the product wage. If the nominal wage ($\omega = P_M w$) is rising faster than food price (

P_F)[8], then the real wage in terms of food $\left(\frac{\omega}{P_A} = \frac{w.P_M}{P_A} = \frac{w}{P}\right)$, in the manufacturing sector will increase. In order to protect their real wages, manufacturing laborers will increase their wages as P increases. I am considering real wage constancy in terms of agricultural price, so real wage in terms of agricultural sector price is: $\Rightarrow \frac{w}{P} = \upsilon$, where υ is constant, i.e., w is perfectly indexed to prices. I know from the dual solution of profit maximization that, $w = MPL$.

$$\Rightarrow w = \delta[\alpha\eta^{\frac{1-\xi}{\xi}} + \delta]^{\frac{1}{\xi-1}}$$

$$\Rightarrow \upsilon P = \delta[\alpha\eta^{\frac{1-\xi}{\xi}} + \delta]^{\frac{1}{\xi-1}} \tag{10}$$

$$\Rightarrow (\frac{\upsilon P}{\delta})^{\xi-1} = \alpha\eta^{\frac{1-\xi}{\xi}} + \delta$$

$$\Rightarrow \eta = [\frac{1}{\alpha}[(\frac{\upsilon P}{\delta})^{\xi-1} - \delta]]^{\frac{\xi}{1-\xi}}$$

where, $\eta = \frac{L}{K_M}$. Again, from equation (10), $\upsilon P = \delta[\alpha\eta^{\frac{1-\xi}{\xi}} + \delta]^{\frac{1}{\xi-1}}$. Binomially expanding equation (10), I get, $\Rightarrow \upsilon P = \delta\left[\binom{\frac{1}{\xi-1}}{0}\left[\alpha\eta^{\frac{1-\xi}{\xi}}\right]^{\frac{1}{\xi-1}}\delta^0 + \binom{\frac{1}{\xi-1}}{1}\left[\alpha\eta^{\frac{1-\xi}{\xi}}\right]^{\frac{1}{\xi-1}-1}\delta^1 + \dots\dots\dots + \delta^{\frac{1}{\xi-1}}\right]$

Taking only the first term and excluding all other terms for the sake of simplification, I get

$$\upsilon P = \delta\alpha^{\frac{1}{\xi-1}}\eta^{-\frac{1}{\xi}}$$

[8] Here, food is used synonymously with agricultural good.

taking the natural log of the above equation and time differentiating, I get,

$$\frac{\dot{P}}{P} = -\frac{1}{\xi}\left(\frac{\dot{\eta}}{\eta}\right) \tag{11}$$

the growth rate of capital in the manufacturing sector is given by

$$\frac{\dot{K}_M}{K_M} = \frac{MP_{K_M} K_M}{Y_M} \frac{Y_M}{K_M} = MP_{K_M} \tag{12}$$

on simplifying, I get[9],

$$\frac{\dot{K}_M}{K_M} = \alpha^{\frac{\xi}{\xi-1}}\left(1-\delta.(\frac{\upsilon P}{\delta})^{1-\xi}\right)^{\frac{1}{1-\xi}} \tag{13}$$

equation (13), is a general mean in 1 and $\left(\frac{\upsilon P}{\delta}\right)$, of order $'1-\xi'$. By the property of general mean, equation (13) will be increasing in $1-\xi$, i.e., decreasing in ξ. Moreover, one can also see from equation (13) that $\frac{\dot{K}_M}{K_M}$ is decreasing in P. Thus, I can conclude that $\frac{\dot{K}_M}{K_M} = f(P,\xi)$ where $f'(P) < 0, f'(\xi) < 0$. Thus, I can write,

$$\frac{\dot{K}_M}{K_M} = aP^{-\xi} \tag{14}$$

which satisfies the above mentioned conditions, where $'a'$ can be interpreted as a productive parameter.

2.3.3 Market Clearing Conditions: The following equations give the demand-supply balance for manufacturing, investment and agricultural sector, respectively.

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[9] For, proof see appendix

$$C_M + Inv. = Y_M \qquad \text{Manufacturing Sector}$$

$$Investment(Inv.) = PY_A + \left(\frac{MP_{K_M} K_M}{Y_M} \right) Y_M \qquad \text{Investment Good}$$

$$PX_A = PY_A$$

$$\Rightarrow \beta w L \left[\frac{1}{wL} \right]^{\varepsilon} [P]^{\gamma - 1} = PY_A \quad ^{10} \qquad \text{Agricultural sector}$$

2.4 DYNAMICS OF THE PRICE PATH

In this section, I deal with the dynamics of the price path and derive certain conditions under which the price path will be convergent. The thrust of this section is to understand the effect of the structural change in the real sector, on the price path. Before, solving for the price path, I briefly relate the specified framework to two cases. First, with $\beta = 0$, the representative household will only consume manufacturing output. This case is the same as that of a one- sector growth model with constant relative risk aversion (CRRA) preference. Second, with $\beta = 0,\ \varepsilon = 0$, I have homothetic preferences and the model abstracts from the income effect of the structural change. As I am interested in the case with non-homothetic preference in a multi-sector model framework, I have imposed the restrictions of $\beta > 0,\ 0 < \varepsilon < 1$.

Dividing the agricultural sector equilibrium throughout by ${K_M}^{1-\varepsilon}$ I get,

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[10] The functional form of the utility function allows us to put the value of X_A only from equation (2), as it represents the total expenditure in agricultural goods.

$$\frac{\beta(wL)^{1-\varepsilon}\left[P\right]^{\gamma-1}}{K_M^{1-\xi}}=\frac{PY_A}{K_M^{1-\xi}}$$
$$\Rightarrow \beta\upsilon^{1-\varepsilon}\eta^{1-\varepsilon}P^{\gamma-\varepsilon}=\frac{PY_A}{K_M^{1-\xi}}$$

taking natural log and time differentiating I get,

$$(1-\varepsilon)\frac{\dot{\eta}}{\eta}+(\gamma-\varepsilon)\frac{\dot{P}}{P}=\bar{x}+\frac{\dot{P}}{P}-(1-\varepsilon)\frac{\dot{K}_M}{K_M}\qquad (\frac{\dot{Y}_A}{Y_A}=\bar{x},\ \text{from equation (9)})$$

substituting the value of $\frac{\dot{\eta}}{\eta}$ and $\frac{\dot{K}_M}{K_M}$, from equation (11) and (14), and simplifying we get,

$$\dot{P}-\left(\frac{\bar{x}}{\Lambda_1}\right)P=-\frac{(1-\varepsilon)a}{\Lambda_1}P^{1-\xi}\tag{15}$$

where $\Lambda_1=\gamma-\varepsilon+\varepsilon\xi-\xi-1$. The above differential equation is a Bernoulli differential equation and has a closed form solution. On solving it, I get,

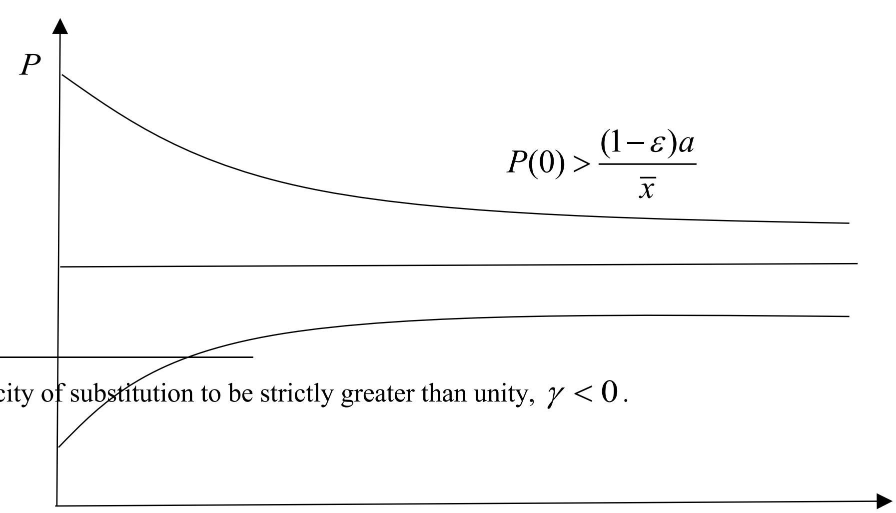
$$P(t)^{\xi}=\left(P(0)^{\xi}-\frac{(1-\varepsilon)a}{\bar{x}}\right)e^{\left(\xi\frac{\bar{x}}{\Lambda_1}\right)t}+\frac{(1-\varepsilon)a}{\bar{x}}\tag{16}$$

The convergence of the price path depends on the sign of the parameter Λ_1. I get a convergent price path if $\Lambda_1<0$ and a divergent price path if $\Lambda_1>0$, as illustrated in Figure 7 and Figure 8, respectively. The sign of Λ_1 is controlled by the strength of income effect and substitution effect and these two effects are controlled by the parameters 'ε' and 'γ'respectively. The empirically relevant case for a developing economy pertains to the fact that agricultural goods are necessary goods. Necessary good implies that expenditure elasticity of agricultural goods '$1-\varepsilon$' should be less than one. The structure of the modeled economy is such that while income effect decreases the expenditure share of the agricultural sector, the substitution effect may increase or decrease the expenditure share of agricultural commodity depending on whether elasticity of substitution between agriculture and manufacturing commodity is less than or greater than unity. Two cases arise out of it; I refer the cases as Case 1 income effect and the substitution effect runs in the same direction and Case 2 when substitution and income effect runs in the opposite direction.

Case1: Income effect and substitution effect runs in the same direction

When the expenditure elasticity of demand is strictly less than unity, and the elasticity of substitution is strictly greater than unity, i.e., the expenditure elasticity of demand of agricultural good '$1-\varepsilon$' is strictly less than unity and the elasticity of substitution in between the agricultural and the manufacturing sector '$1-\gamma+(\varepsilon-\gamma)\left(\frac{S_A}{1-S_A}\right)$' is strictly greater than unity. In such a case the price path given by equation (16) will be stable/convergent as $\Lambda_1<0$[11] and the equilibrium or steady state price will also be low because $0<1-\varepsilon<1$, as shown in Figure 7 below.

*Figure **Error! No text of specified style in document.**-7: Convergent price path*



[11] For elasticity of substitution to be strictly greater than unity, $\gamma<0$.

$$\overline{P} = \frac{(1-\varepsilon)a}{\overline{x}}$$

$$P(0) < \frac{(1-\varepsilon)a}{\overline{x}}$$

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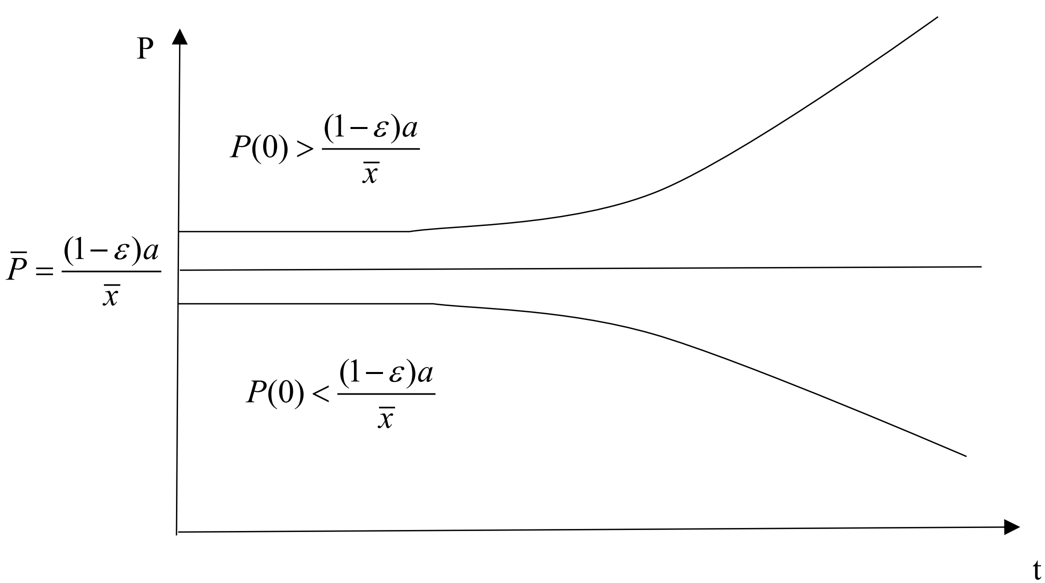
Figure ***Error! No text of specified style in document.****-8: Divergent price path*



Case 2: Income effect and the substitution effect are running in opposite direction

There are two subcases in it: Case 2.1 when substitution effect dominates income effect and Case 2.2 when income effect dominates substitution effect.

Case 2.1: Substitution effect dominates the income effect, i.e., $1-\varepsilon < 1-\gamma+(\varepsilon-\gamma)\left(\frac{S_A}{1-S_A}\right)$

Both the expenditure elasticity of demand is strictly less than unity, and the elasticity of substitution is strictly less than unity, i.e., when the expenditure elasticity of demand of agricultural good '$1-\varepsilon$' is strictly less than unity, and the elasticity of substitution in between the agricultural and the manufacturing sector '$1-\gamma+(\varepsilon-\gamma)\left(\frac{S_A}{1-S_A}\right)$' is strictly less than unity. Moreover, as substitution effect dominates income effect $\gamma-\varepsilon < (\varepsilon-\gamma)\left(\frac{S_A}{1-S_A}\right)$, which is negative. This case is not possible as $\gamma-\varepsilon > 0$, from the conditions of elasticity of substitution being strictly less than unity. So, substitution effect will never dominate income effect.

Case 2.2: Income effect dominates the substitution effect, i.e., $1-\varepsilon > 1-\gamma+(\varepsilon-\gamma)\left(\frac{S_A}{1-S_A}\right)$

This is same as the above mentioned case, the only difference being $\gamma-\varepsilon > (\varepsilon-\gamma)\left(\frac{S_A}{1-S_A}\right)$, which allows us the flexibility for $\gamma-\varepsilon > 0$. In such a case, the price path given by equation (16) will be stable/convergent as $\Lambda_1 < 0$[12]. Graphically, it is same as Figure 3 above. Thus I see that irrespective of the direction of movement of income effect and substitution effect, I get a value of $\Lambda_1 < 0$, which implies a convergent price path.

2.4.1 Properties of the Price Path: There are four important of the price path:

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[12] As $0 < \gamma-\varepsilon < 1$, and the other components of Λ_1 being less than negative one, I can safely say $\Lambda_1 < 0$

I. The relevant case for the Indian economy as is evident from Figure 3 and Figure 4, is when the income and the substitution effect is running in the opposite direction for the income inelastic sector, but the income effect outweighs the substitution effect. In such a case the price path given by equation (16) will be stable/convergent as $\Lambda_1 < 0$[13] and the equilibrium or steady state price will be low because $0 < 1 - \varepsilon < 1$, as shown in Figure 7.

II. The initial share of agricultural sector in any developing economy is always larger than the manufacturing sector as is evident from Figure 3. The relative price path in Figure 4, also shows an increasing trend. Thus, combining Figure 3 and Figure 4, I can safely assume that the initial price $P(0)$ is less than $\overline{P}$ and it increases over time until it reaches $\overline{P}$.

III. In the long-run, sectoral growth rates are not balanced. I can see this by comparing equation (16) with the growth rates of the agricultural and manufacturing sector. I have assumed the growth rate of agriculture (g_A) as $\overline{x}$. The manufacturing sector production function is constant returns to scale (CRS) in nature so I can say that the manufacturing sector growth rate is equal to $\frac{\dot{K}_M}{K_M}$, which equals $a\overline{P}^{-\xi}$, but there are certain caveats. Ideally, in this model $\frac{K_M}{L}$, is not constant, and as the wage is perfectly indexed to prices, the growth rate of manufacturing sector output cannot be equal to the growth of capital in the manufacturing sector. However, in the model, the initial price $(P(0))$ and the long-run or steady state price is fixed ($P = \overline{P}$), so, in the initial phase manufacturing growth rate will be equal to the growth rate of capital in the manufacturing sector. In the long-run when $P = \overline{P}$, because the price is fixed I can again safely assume that the manufacturing sector

[13] For elasticity of substitution to be strictly less than unity and $\Lambda_1 < 0$, I need $-1 < \gamma < 0$, and as $0 < \varepsilon < 1$, and, $0 < \xi < 1$, so $0 < \varepsilon\xi < 1$, which is less than one, so Λ_1 will be less than zero.

growth rate will be equal to the capital sector growth rate which is equal to the employment growth rate. For balanced growth both the sector should grow at the same rate, but using equation (16), I find that $\bar{x} = (1-\varepsilon)a\bar{P}^{-\xi} < a\bar{P}^{-\xi}$, as $0 < 1-\varepsilon < 1$. Thus, structural change is an asymptotic phenomenon in the model because of non-homotheticity of preference. The relative price act as an intertemporal equilibrating variable. The prices keep on increasing as long as the growth gap between agricultural sector and manufacturing sector prevails.

IV. In point 3 above, I commented on the structural change process after the relative price has reached its long-run value and proved that structural change is an asymptotic phenomenon in developing economies. Moreover, I also illustrated how relative price act as an intertemporal equilibrating variable when there is unbalanced growth across sectors. As I am interested in the long-run equilibrium value, so I can equate $\frac{\dot{Y}_M}{Y_M} = \frac{\dot{K}_M}{K_M}$. However, in the model real wage is fixed, as a result, as price changes so do nominal wage, so does $\frac{K_M}{L}$ and I am unable to equate $\frac{\dot{Y}_M}{Y_M} = \frac{\dot{K}_M}{K_M}$ in the transitory phase of the economy. In this point I further on the intertemporal equilibrating role of relative price in the transition phase and comment on the extent of increase in relative prices due to the difference in growth across sectors through the parameter, elasticity of substitution between factors of production, ξ. Because it is difficult to measure growth rates during the transition phase, I use economic reasoning instead of mathematical proof wherever required.

The elasticity of substitution between factors of production in the manufacturing sector also has a strong influence on sector growth rates and structural change. The range of values for ξ lies in the interval $[0, \infty)$. I divide this interval into two regions $\xi < 1$ and $\xi > 1$. For $\xi < 1$, capital and labor are complementary to each other whereas for $\xi > 1$, capital and labor becomes increasingly substitutable. A word of caution, although I have not taken any technology parameter explicitly in the manufacturing production function, ξ itself acts as a technology parameter (Klump & Grandville, 2000). There exists a threshold value for ξ corresponding to which I get a permanent increase in per capita output growth as well as per capita income growth. The calculation of the threshold value is not within the scope of this paper. Instead, I show that the per capita output in the manufacturing sector is an increasing function of ξ. Rewriting equation (8) in per capita output form I get, $\frac{Y_M}{L} = \left[\alpha \left(\frac{K_M}{L} \right)^{\frac{\xi-1}{\xi}} + \delta \right]^{\frac{\xi}{\xi-1}}$, which is again a general mean in $\frac{K_M}{L}$ and 1 of order $\frac{\xi-1}{\xi}$, where $\frac{\xi-1}{\xi}$ is increasing in ξ. Thus, per capita output increases as ξ increases, but when $0 < \xi < 1$, per capita output decreases as ξ increases because within this range $\frac{\xi-1}{\xi}$ is decreasing in ξ. Similarly, when $\xi > 1$ one can see that per capita output increases as ξ increases. Moving on to the price path as given is equation (16), I get two cases when $\xi < 1$, and $\xi > 1$. When $\xi > 1$, the manufacturing sector growth slows down, and the growth gap between the manufacturing and agricultural sector decreases during the transition resulting in slowing down of relative prices, i.e., at levels, the relative price will decrease, but when $\xi > 1$, using the same strand of logic the growth gap enhances and accelerates the increases in relative prices resulting in higher levels of inflation.

2.4.2 Discussion about Inflation Targeting: Inflation targeting is a monetary policy regime wherein the central bank targets a specific level of inflation to be achieved within a stipulated time period using some policy variable specifically the interest rate to target it. I see from the model I have developed that the relative price acts as an intertemporal equilibrating variable to balance the growing gap between the sectors. Specifically, from equation (16), if I do some comparative static exercise, I find that

1. A one shot rise in the agriculture productivity by increasing $\bar{x}$ will result in a decrease in the steady state price $P = \bar{P}$. As a result, not only the level of prices will decrease, but the slope of the transition path will also become a bit flatter, implying a lower level of inflation in the short to medium term.
2. A one shot rise in manufacturing sector growth rate through an increase in a will result in increasing the growth gap. It will also increase the steady state level of prices, and the slope of the transition path of the relative price will become steeper, implying a higher level of inflation in the short to medium term.

If I discuss the above mentioned comparative static exercise with regard to glide path, then with an increase in $\bar{x}$, the level of price decreases and the glide path becomes flatter. It implies a decreasing level of inflation whereas when I increase the manufacturing sector growth rate the glide path gets steeper implying an increase in the level of inflation. The reason why I am getting such results is that the agricultural sector and the manufacturing sector are gross complement which is also the empirical relevant case, and the growth of the economy is determined by the growth of the slowest growing sector. The relevant policy tool should be to focus investment in the agricultural sector so as to increase growth and lower inflation.

2.5 KALDOR & KUZNETS FACT

2.5.1 Structural Change and Kuznets Fact: Structural change is immediate by property 4.1.III. At the initial price $P(0)$, there is a growth gap. Over time the growth gap reduces because of increase in relative price. As the growth gap closes, $\left(\frac{Y_M}{Y_A}\right)$ ratio increases over time. This is nothing but the structural change. Formally I denote structural change (θ) as $\theta = \frac{Y_M}{Y_A}$

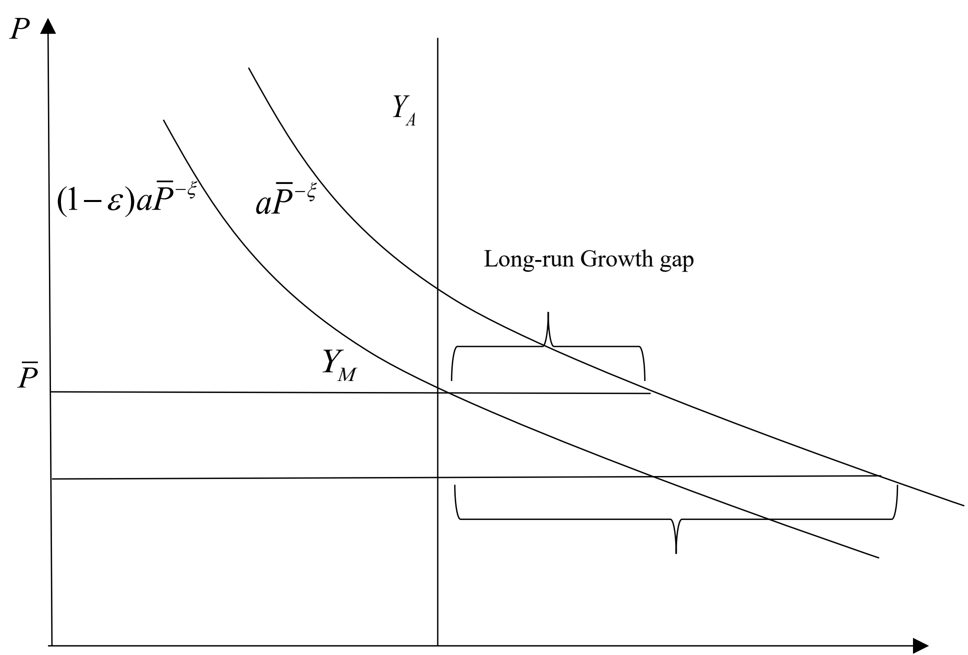
$$\Rightarrow \frac{\dot{\theta}}{\theta} = \frac{\dot{Y}_M}{Y_M} - \frac{\dot{Y}_A}{Y_A} = a\overline{P}^{-\xi} - \overline{x} \tag{17}$$

$$\Rightarrow \frac{\dot{\theta}}{\theta} = \overline{x}\left(\frac{\varepsilon}{1-\varepsilon}\right) \tag{18}$$

$$\Rightarrow \theta(t) = \theta_{\overline{P}} e^{\overline{x}\left(\frac{\varepsilon}{1-\varepsilon}\right)}$$

Equation (17) shows the equation of motion of structural change after $\overline{P}$ has reached. There will be no structural change, only when $\varepsilon = 0$, i.e., the income elasticity of an agricultural good is one. Figure 9 illustrates the result graphically. In the diagram, I have assumed that $P(0) < \overline{P}$, because the output of the Y_M sector is low relative to the marketed surplus which is true for developing economies at the early stages of development.

Figure ***Error! No text of specified style in document.****-9: Structural change*



$P(0)$

Initial Growth gap

$\bar{x}$ t

2.5.2 Aggregate Dynamics and Kaldor Facts: In this section, I will discuss the Kaldor facts and the aggregate dynamics of the model. Kaldor facts are a list of stylized facts which characterize the growth process across countries. I kept this section at the end because I needed to know the exact path of the disaggregated variables in the model. The dynamic equilibrium path that this model is going to follow is determined by the relative prices specifically the initial and long-run relative price. As I have shown in section 2.4 where I derive the dynamics of the price path, the case best suited for a developing economy is a case where the agricultural sector output is a necessity good or inelastic in nature, and as the relative price of the agriculture output is increasing, its expenditure share is decreasing. The dynamic equilibrium path is determined by the fact that initial price is $P(0) < \bar{P}$ and $\bar{P} = \frac{(1-\varepsilon)a}{\bar{x}}$, which are all parameters in the model. After the dynamic equilibrium path is determined, I determine the wage rate and the interest rate in the economy in the next sub-section, which will help us in determining the aggregate dynamics of the model.

Wage rate and rate of return on capital before/after $\bar{P}$: It is known, $r = MP_{K_M} = \alpha\left(\frac{Y_M}{K_M}\right)^{\frac{1}{\xi}}$.

Using equation (14), I can say,

$$r = aP^{-\xi} \tag{19}$$

from the assumption of constant real wage rate in terms of the relative price of agriculture, I can write,

$$w = \upsilon P \tag{20}$$

in the initial phase of development, the agricultural sector has a larger output share compared to the manufacturing sector[14]. So from equation (20), before price reaches $\overline{P}$, the manufacturing product wage is increasing, whereas from equation (19), the rate of return on capital is decreasing. In long-run, at $P = \overline{P}$, manufacturing product wage and rate of return on capital is constant and is equal to, $a\overline{P}^{-\xi}$ and $\upsilon\overline{P}$, respectively.

Aggregate Dynamics: I have already shown the disaggregate equilibrium in the previous section and the transition path of the economy through the dynamics of price path and structural change. In this section, I focus on the asymptotic equilibrium paths which are equilibrium paths as $t \to \infty$. A constant growth path (CGP) is defined as an equilibrium path where the asymptotic growth rate of expenditure on consumption exists and is constant, i.e., $\lim_{t\to\infty}\frac{\dot{E}}{E} = g^*$

From the Euler equation (6) this also implies that the interest rate must be asymptotically constant, i.e., $\lim_{t\to\infty}\dot{r} = 0$. To establish the existence of constant CGP, I impose the following parameter restriction, i.e., $r^* > \rho$, where $r*$ is the constant asymptotic interest rate. In the long-run, from equation (6) and equation (19), the expenditure share, is growing at a constant rate of

$$\frac{\dot{E}(t)}{E(t)} = \frac{1}{(1-\varepsilon)}\left[\left(a\overline{P}^{-\xi}\right) - \rho\right]$$

[14] From equation (16), the initial price $P(0) < \overline{P}$.

and in the long-run, using equation (19), the interest rate is constant and is equal to, $r = a\overline{P}^{-\xi}$. Moreover, in the long-run, using equation (20), it is seen that the wage rate, $w = \upsilon\overline{P}$, and the wage rate grows pari passu with the growth rate of prices. Thus, although I characterize a CGP, that aggregate expenditure grows at a constant rate, but growth is essentially non-balanced as sectoral outputs grow at different rates.

2.6 ELASTICITY OF SUBSTITUTION AND ITS IMPACT

In this section, I will elaborate on property 4.1.IV and check the effect of a change in elasticity of substitution between capital and labor (ξ) on the price path given by equation (16). Following, Pitchford (1960) I first define the limiting values of $\frac{Y_M}{K_M}$.This ratio is important because it defines the growth rate of K_M (refer to equation (12)). Following, Grandville and Solow (2009) approach and using equation (8), the limiting values are defined as follows:

$$\lim_{\frac{K_M}{L}\to\infty}\left(\frac{Y_M}{K_M}\right) = \inf\frac{Y_M}{K_M} = \alpha^{\frac{\xi}{\xi-1}},\ \xi>1 \tag{21}$$

$$\lim_{\frac{K_M}{L}\to 0}\left(\frac{Y_M}{K_M}\right) = \sup\frac{Y_M}{K_M} = \alpha^{\frac{\xi}{\xi-1}},\ \xi<1 \tag{22}$$

Rewriting equation (8) in the form of $\frac{K_M}{Y}$ and $\frac{L}{Y}$, I write it in the manner shown in equation (23)

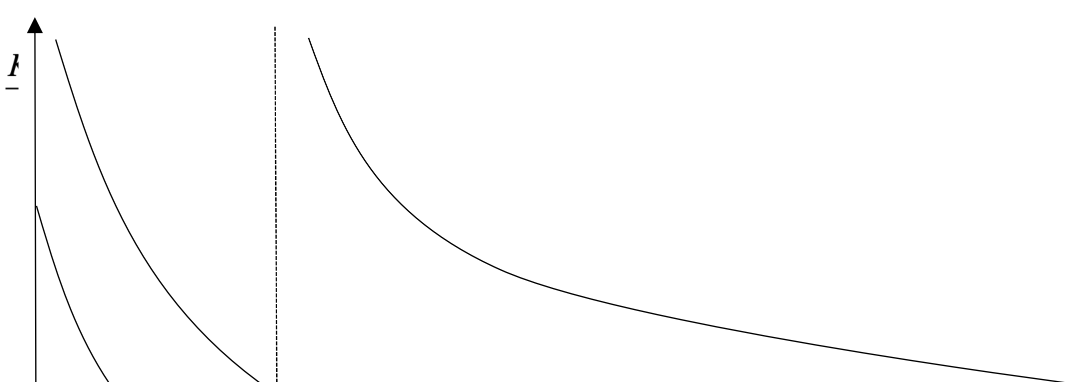
$$\frac{K}{Y_M} = \left[\frac{1}{\alpha} - \frac{\delta}{\alpha}\left(\frac{L}{Y_M}\right)\right]^{\frac{\xi}{\xi-1}} \tag{23}$$

for graphical ease as can be seen in Figure 10. Combining equation (13), equation (22), and equation (23), I can write that in the steady state,

$$\frac{\dot{K}_M}{K_M} = \alpha^{\frac{\xi}{\xi-1}} \qquad (24)$$

Figure 10, shows the shape of isoquants with different values of elasticity of substitution. I have superimposed the limiting condition in this diagram to show the steady state or long-run growth values that can be achieved by the isoquants with differing elasticity of substitution specifically with ξ greater than, equal to or less than one. From Figure 10, it is clear that for any value of $\left(\frac{K_M}{Y_M}, \frac{L}{Y_M}\right)$ the economy can reach any steady state with $\xi = 1$(Cobb-Douglas Production function). However, for $\xi < 1$, the economy cannot reach any high steady state value of $\frac{\dot{K}_M}{K_M}$ because of the limit imposed by equation (22), as shown by the $\left(\frac{\bar{K}_M}{\bar{Y}_M}\right)$ line in Figure 10, whereas for $\xi > 1$, the economy will be able to reach higher steady state values of $\frac{\dot{K}_M}{K_M}$, as is demonstrated by the point $\frac{\bar{K}_M}{\bar{Y}_M}$, in the above diagram. Moreover, in the limiting case of equation (24), $\frac{\dot{K}_M}{K_M}$ increases if ξ increases. Thus one can see that with $\xi > 1$, the growth rate in the manufacturing sector increases. To summarize, there are two cases, when $\xi < 1$, the maximum rate at which the manufacturing sector grows is $\alpha^{\frac{\xi}{\xi-1}}$, and growth rate of manufacturing sector will decrease with

*Figure **Error! No text of specified style in document.**-10: Elasticity of substitution and its impact*



increase in ξ. Whereas when $\xi > 1$, the minimum rate at which the manufacturing sector will grow is $\alpha^{\frac{\xi}{\xi-1}}$, and the growth rate will increase with an increase in ξ. I have used the limiting values of $\frac{Y_M}{K_M}$, when analyzing the impact of ξ on structural change and not on the price path, because structural change is an asymptotic phenomenon in the model.

2.6.1 Impact on Price Equation: ξ affects the price equation at the level as well as in the rate of change of prices. I will first deal with the level changes. From equation (16), I can write the steady state level of prices as,

$$\bar{P} = \left[\frac{(1-\varepsilon)a}{\bar{x}}\right]^{\frac{1}{\xi}} \tag{25}$$

equation (25) clearly shows that as ξ increases, the level of prices decreases. The rate of change of price is also determined by the ξ present in the exponential term of equation (16), and it is quite evident that as ξ increases, the rate of growth of prices will also increase because ξ affects the steepness of the price path.

2.6.2 Impact on Structural Change: As I have already proved before that structural change is an asymptotic phenomenon in the model. In this section when I talk about structural change, I mean the growth gap between the agricultural and manufacturing sector, and by the impact on structural change, I mean whether the growth gap is increasing or decreasing.

I have found that the asymptotic growth rate of the manufacturing sector is $\alpha^{\frac{\xi}{\xi-1}}$. In the steady state combining equation (14) and equation (24), I can write,

$$\frac{\dot{K}_M}{K_M} = a\overline{P}^{-\xi} = \alpha^{\frac{\xi}{\xi-1}} \tag{26}$$

when there is balanced growth, g_A and g_M should be equal. There are two cases.

In the first case, when $0 < \xi < 1$, from equation (26), as ξ increases, the growth rate of manufacturing sector will decrease, which from equation (17), will decrease the rate of structural change.

In the second case, when $\xi > 1$, from equation (26), the growth rate of manufacturing sector will increase, which from equation (17), will increase the rate of structural change.

2.7 CONCLUSION

This paper presents a model which shows the impact of the real sector on the relative price dynamics in a developing economy from both the supply side and demand side, with observations for India in the background. The only theoretical contribution in this area is that of Boppart (2014). Boppart (2014), presents a parsimonious growth model, which is consistent with structural change, relative price dynamics and Kaldor facts using a neoclassical production function. However, in developing economies, all the sectors do not have standard neoclassical production function with smooth factor substitution. Our model contributes to this area by assuming a CES production function. I also show the transition dynamics in the model through the behavior of relative prices which acts as the intertemporal equilibrating variable, and is absent in Boppart (2014).

The result of the model shows that relative price path is related to the equilibrium dynamics arising out of sectoral differences in production structure and their growth as well as the changing demand composition. In the long-run, sectoral growths remain unbalanced due to non-unitary income elasticities. Structural change is an outcome of closing this growth gap over time. The closing of growth gap depends on how sectoral growth rates respond to relative price changes. Moreover, one can also see that an increase in elasticity of substitution between the factors of production specifically, when $0 < \xi < 1$, may reduce the extent of structural change, i.e., decrease the growth gap between the sectors in the long-run, but will inevitably result in higher level of prices as well as the rate of growth of prices.

The policy implication of this paper goes much beyond; inflation targeting has become the norm of monetary policy throughout the world, but developing economies unlike developed economies, as have been illustrated in the motivating example section, are going through a phase wherein structural change seems to be a persistent phenomenon. In developing economies, the agricultural sector has a large share, to begin with, and the model shows how such initial imbalance gives rise to an eventual increase in the relative price of agriculture and accompanying structural change. Since price change is a property of long-run growth, the study of inflation must go beyond the usual short-run analysis of output-inflation trade-off. The short-run policy must, therefore, be consistent growth path of the real sectors, driven by the underlying economic structure. Our finding is that the dynamics of the real sector play a paramount role in changes in relative price and thus in structural change. Policies that attempt to rectify sectoral imbalances are as important as monetary policies that tackle inflation in the developing economies.

One can also see that to address the relevant case wherein elasticity of substitution in between manufacturing and agricultural sector is less than one requires the presence of income effect to explain the pattern in the data. In this paper, I have developed a tractable structural change model for developing economies which incorporates both the asymptotic nature of structural change and non-homothetic preference exclusively focusing on the price path and its out of steady state behavior.

This paper also addresses the issue of inflation targeting in developing economies and finds that although inflation targeting is a relevant policy weapon, for developing economies it needs to be taken with a pinch of salt. The policy decision that I conclude from the theoretical model is that it's the slowest growing sector that determines the growth of an economy and to increase growth and decrease inflation, one need to focus investment in the slowest growing sector (agricultural sector.)

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APPENDIX

Proof of Equation (2), (3), (4), (5)

PROOF: Equation (1) is an indirect utility function. To derive the demand functions for the agricultural and manufacturing good, I rewrite equation (1), without normalizing the price of the manufacturing sector to one, so equation (1), becomes $V(.) = \frac{1}{\varepsilon}\left(\frac{E}{P_M}\right)^{\varepsilon} - \frac{\beta}{\gamma}\left(\frac{P_A}{P_M}\right)^{\gamma} - \frac{1}{\varepsilon} + \frac{\beta}{\gamma}$, then I use Roy's identity that states $X_j = -\frac{\partial V(.)/\partial P_j}{\partial V(.)/\partial E}$, $\forall j = A, M$. I have $\partial V(.)/\partial P_A$ $= -\beta P_A^{\gamma-1} P_M^{-\gamma}$, $\partial V(.)/\partial P_M = -E^{\varepsilon} P_M^{-\varepsilon-1} + \beta P_A^{\gamma} P_M^{-\gamma-1}$, and $\partial V(.)/\partial E = E^{\varepsilon-1} P_M^{-\varepsilon}$. Applying Roy's identity, and normalizing the price of manufacturing good to one I will get equation (2) and equation (3). Dividing equation (2) and equation (3), throughout by E, I will get equation (4) and equation (5). Q.E.D.

Proof of Equation (12)

PROOF: From equation (12), I know the growth rate of capital in manufacturing sector is $\frac{\dot{K}_M}{K_M} = MP_{K_M}$. Rewriting equation (12) in the form of the per capita capital production I get,

$$\frac{\dot{K}_M}{K_M} = \alpha\left[\frac{Y_M}{K_M}\right]^{\frac{1}{\xi}} = \alpha\eta^{\frac{1}{\xi}}\left[\alpha\eta^{\frac{1-\xi}{\xi}} + \delta\right]^{\frac{1}{\xi-1}} = \alpha\eta^{\frac{1}{\xi}}\left(\frac{\upsilon P}{\delta}\right) = \alpha^{\frac{\xi}{\xi-1}}\left[\left(\frac{\upsilon P}{\delta}\right)^{1-\xi}\left(\left(\frac{\upsilon P}{\delta}\right)^{\xi-1} - \delta\right)\right]^{\frac{1}{1-\xi}} =$$

$$\alpha^{\frac{\xi}{\xi-1}}\left(1 - \delta.(\frac{\upsilon P}{\delta})^{1-\xi}\right)^{\frac{1}{1-\xi}}$$ Q.E.D